EGT2
ENGINEERING TRIPOS PART IIA
EGT3
ENGINEERING TRIPOS PART IIB

Friday 1st May 2026   14.00 to 15.40

Module 4M16

NUCLEAR POWER ENGINEERING

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 4M16 Nuclear Power Engineering data sheet (8 pages)
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

1 Consider a research reactor used to understand the behaviour of small power reactors. During operation, neutron flux measurements are required and radioactive materials must be handled by staff.

An activation foil is inserted to measure the flux in the reactor. The foil contains a pure isotope (with number density N_{p}) which activates (with microscopic cross section σ_{a}) to become radioactive. The product isotope decays with a decay constant of λ, emitting a gamma ray of energy E_γ, but does not measurably interact with neutrons.

(a) Deduce a differential equation governing the number density of the product isotope and, further, show that after some time t the flux can be measured as

$$\phi = \frac{A(t)}{\sigma_{\text{a}} N_{\text{p}} \left(1 - \mathrm{e}^{-\lambda t}\right)}$$

where $A(t)$ is the activity per unit volume of the product isotope. Assume that N_{p} does not change significantly over time. [30%]

(b) After irradiation for five hours, the foil is removed from the reactor and allowed to decay for one hour before measuring its activity. Assume the foil can be treated as a point source with a 1 cm^3 volume. The dose in Sv from a constant activity gamma source can be estimated using the following formula

$$D = \frac{1.6 \times 10^{-13} \Sigma E_\gamma A t}{4\pi \rho r^2}$$

where Σ is the macroscopic cross section for gamma rays in human tissue in m^{-1}, E_γ is the gamma energy in MeV, A is the activity in Bq, t is the time in seconds, ρ is the density of human tissue in kg/m^3, and r is the distance in metres.

(i) Assuming the foil's activity remains constant during the measurement process, estimate the dose (in Sv) to staff working at a distance of 2 m from the foil for 20 minutes. [30%]

(ii) Compute the dose if we discard the assumption of a constant activity. Under which circumstance is the assumption of constant activity a good one? [25%]

(c) Explain how the principles of radiation protection should be applied in this scenario to limit the dose to staff. [15%]

Data: $\sigma_{\text{a}} = 5$ b, $N_{\text{p}} = 5 \times 10^{21}$ cm^{-3}, $\lambda = 0.693$ hr^{-1}, $\phi = 10^{13}$ cm^{-2}s^{-1}, $E_\gamma = 1$ MeV, $\Sigma = 1$ m^{-1}, $\rho = 1000$ kg m^{-3}

2 For the purpose of analysis, a nuclear reactor can often be considered as a repeating lattice of fuel and moderator. This is usually modelled by applying reflective boundary conditions $\left(\frac{d\phi}{dx} = 0\right)$ to the centres of the repeating lattice materials.

Consider a single repeating period of fuel and moderator (illustrated in Fig. 1); the fuel region has a half-width of a and the moderator region has a half-width of b. The fuel has a buckling of B (with $\eta > 1$) and the moderator has a diffusion length of L.

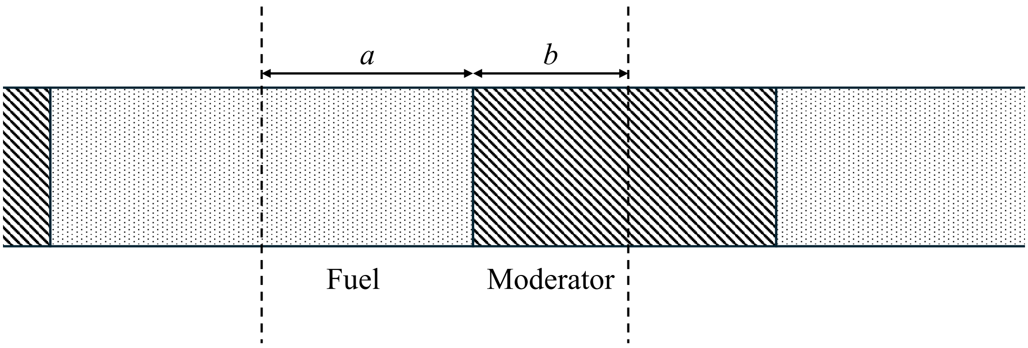

Fig. 1: Illustration of a periodic fuel/moderator lattice.

(a) For a fuel slab centred at $x = 0$ and a moderator slab centred at $x = a + b$, show that within the fuel region the flux solution is of the form:

$$\phi_{\text{fuel}}(x) = A\cos(Bx)$$

and, for the moderator:

$$\phi_{\text{mod}}(x) = C\cosh\left(\frac{x - a - b}{L}\right)$$

where A and C are arbitrary constants. Assume that monoenergetic neutron diffusion theory applies. [40%]

(b) Show that the system has the following criticality condition:

$$D_{\text{fuel}} B\tan(Ba) = \frac{D_{\text{mod}}}{L}\tanh\left(\frac{b}{L}\right)$$

where D_{fuel} and D_{mod} are the diffusion coefficients of the fuel and moderator, respectively. Note that at the boundaries of different material regions the flux, ϕ, and the current, $J = -D\frac{d\phi}{dx}$, must be continuous. [50%]

(c) Why is the chosen model of neutron behaviour a poor approximation in this system? [10%]

3 The single precursor group point kinetics equations are:

$$\frac{\mathrm{d}n}{\mathrm{d}t} = \frac{\rho - \beta}{\Lambda} n + \lambda_{\mathrm{d}} c$$

$$\frac{\mathrm{d}c}{\mathrm{d}t} = \frac{\beta}{\Lambda} n - \lambda_{\mathrm{d}} c$$

(a) Explain the meaning of each term in the equations. You are not required to explain the meaning of individual symbols. [20%]

(b) A new fissile isotope is discovered which, on fission, does not directly produce a precursor. Instead, with the same probability β as above, it produces a *pre-precursor* on fission (an isotope which decays to produce a delayed neutron precursor). The decay constant of this pre-precursor is λ_{p}. The remaining fraction of fission neutrons, $(1 - \beta)$, are prompt.

(i) Assuming that pre-precursors have a negligible neutron cross section, show that their governing equation is:

$$\frac{\mathrm{d}p}{\mathrm{d}t} = \frac{\beta}{\Lambda} n - \lambda_{\mathrm{p}} p$$

where p is the pre-precursor population, and explain how the point kinetics equations must be modified to accommodate them. [20%]

(ii) By assuming an exponential solution or otherwise, show that the inhour equation for a reactor fuelled by this new fissile isotope is:

$$\rho = \Lambda\omega + \beta - \frac{\beta\lambda_{\mathrm{p}}\lambda_{\mathrm{d}}}{(\lambda_{\mathrm{p}} + \omega)(\lambda_{\mathrm{d}} + \omega)}$$

where ω is the inverse period of the reactor. [50%]

(iii) How will the value of λ_{p} affect reactor control relative to a conventional system, assuming that λ_{d} and β remain unchanged? [10%]

4 (a) Describe four ways of treating gaseous, liquid and solid radioactive wastes, giving the advantages and disadvantages of each process in terms of applicability, cost and operator dose uptake. [60%]

(b) One of the waste streams arising is a bleed from spent fuel storage ponds which contains, amongst other nuclides, Ag-110m. This stream is passed through an ion exchange plant before further treatment, and then discharged.

(i) Using the data provided, calculate the activity of Ag-110m leaving the ion exchange plant. Ignore any decay during passage through the medium. [10%]

(ii) If the ion exchange medium is replaced and sent for disposal every two years, what will be the total activity in Bq and the concentration of activity in Bq/g of Ag-110m on the discharged medium? Assume the medium is discharged as soon as the effluent flow is stopped. [30%]

Data:

Ag-110m decays to Cd-110 with a half-life of 252 days

Flow rate: 50 m^3/hr

Density of effluent: 1000 kg/m^3

Inlet activity: 10 Bq/g

Decontamination factor: 20

Volume of ion exchange medium: 500 m^3

Density of ion exchange medium: 1200 kg/m^3

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MODULE 4M16

NUCLEAR POWER ENGINEERING

DATA SHEET

General Data

Speed of light in vacuum	c	$299.792458 \times 10^{6}\ \mathrm{m\,s^{-1}}$
Magnetic permeability in vacuum	μ_0	$4\pi \times 10^{-7}\ \mathrm{H\,m^{-1}}$
Planck constant	h	$6.62606957 \times 10^{-34}\ \mathrm{J\,s}$
Boltzmann constant	k	$1.380662 \times 10^{-23}\ \mathrm{J\,K^{-1}}$
Elementary charge	e	$1.6021892 \times 10^{-19}\ \mathrm{C}$

Definitions

Unified atomic mass constant	u	$1.6605655 \times 10^{-27}\ \mathrm{kg}$ $(931.5016\ \mathrm{MeV})$
Electron volt	eV	$1.6021892 \times 10^{-19}\ \mathrm{J}$
Curie	Ci	$3.7 \times 10^{10}\ \mathrm{Bq}$
Barn	barn	$10^{-28}\ \mathrm{m^2}$

Atomic Masses and Naturally Occurring Isotopic Abundances (%)

	electron	0.00055 u	90.80%	$^{20}_{10}Ne$	19.99244 u
	neutron	1.00867 u	0.26%	$^{21}_{10}Ne$	20.99385 u
99.985%	$^{1}_{1}H$	1.00783 u	8.94%	$^{22}_{10}Ne$	21.99138 u
0.015%	$^{2}_{1}H$	2.01410 u	10.1%	$^{25}_{12}Mg$	24.98584 u
0%	$^{3}_{1}H$	3.01605 u	11.1%	$^{26}_{12}Mg$	25.98259 u
0.0001%	$^{3}_{2}He$	3.01603 u	0%	$^{32}_{15}P$	31.97391 u
99.9999%	$^{4}_{2}He$	4.00260 u	96.0%	$^{32}_{16}S$	31.97207 u
7.5%	$^{6}_{3}Li$	6.01513 u	0%	$^{60}_{27}Co$	59.93381 u
92.5%	$^{7}_{3}Li$	7.01601 u	26.2%	$^{60}_{28}Ni$	59.93078 u
0%	$^{8}_{4}Be$	8.00531 u	0%	$^{87}_{35}Br$	86.92196 u
100%	$^{9}_{4}Be$	9.01219 u	0%	$^{86}_{36}Kr$	85.91062 u
18.7%	$^{10}_{5}B$	10.01294 u	17.5%	$^{87}_{36}Kr$	86.91337 u
0%	$^{11}_{6}C$	11.01143 u	12.3%	$^{113}_{48}Cd$	112.90461 u
98.89%	$^{12}_{6}C$	12.00000 u		$^{226}_{88}Ra$	226.02536 u
1.11%	$^{13}_{6}C$	13.00335 u		$^{230}_{90}Th$	230.03308 u
0%	$^{13}_{7}N$	13.00574 u	0.72%	$^{235}_{92}U$	235.04393 u
99.63%	$^{14}_{7}N$	14.00307 u	0%	$^{236}_{92}U$	236.04573 u
0%	$^{14}_{8}O$	14.00860 u	99.28%	$^{238}_{92}U$	238.05076 u
99.76%	$^{16}_{8}O$	15.99491 u	0%	$^{239}_{92}U$	239.05432 u
0.04%	$^{17}_{8}O$	16.99913 u		$^{239}_{93}Np$	239.05294 u
0.20%	$^{18}_{8}O$	17.99916 u		$^{239}_{94}Pu$	239.05216 u
				$^{240}_{94}Pu$	240.05397 u

Simplified Disintegration Patterns

Isotope	${}^{60}_{27}Co$	${}^{90}_{38}Sr$	${}^{90}_{39}Y$	${}^{137}_{55}Cs$	${}^{204}_{81}Tl$
Type of decay	β^-	β^-	β^-	β^-	β^-
Half life	5.3 yr	28 yr	64 h	30 yr	3.9 yr
Total energy	2.8 MeV	0.54 MeV	2.27 MeV	1.18 MeV	0.77 MeV
Maximum β energy	0.3 MeV (100%)	0.54 MeV (100%)	2.27 MeV (100%)	0.52 MeV (96%) 1.18 MeV (4%)	0.77 MeV (100%)
γ energies	1.17 MeV (100%) 1.33 MeV (100%)	None	None	0.66 MeV (96%)	None

Thermal Neutron Cross-sections (in barns)

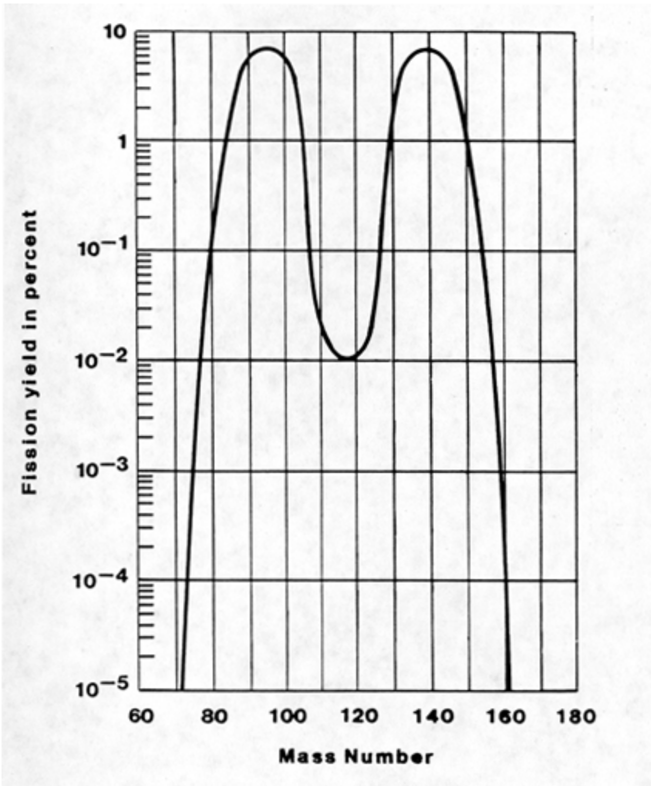
	"Nuclear" graphite	${}^{16}_{8}O$	${}^{113}_{48}Cd$	${}^{235}_{92}U$	${}^{238}_{92}U$	${}^{1}_{1}H$ unbound
Fission	0	0	0	580	0	0
Capture	4×10^{-3}	10^{-4}	27×10^{3}	107	2.75	0.332
Elastic scatter	4.7	4.2		10	8.3	38

Densities and Mean Atomic Weights

	"Nuclear" graphite	Aluminium Al	Cadmium Cd	Gold Au	Uranium U
Density / $kg\,m^{-3}$	1600	2700	8600	19000	18900
Atomic weight	12	27	112.4	196	238

Fission Product Yield

Nuclei with mass numbers from 72 to 158 have been identified, but the most probable split is unsymmetrical, into a nucleus with a mass number of about 138 and a second nucleus that has a mass number between about 95 and 99, depending on the target.



The primary fission products decay by β^- emission. Some important decay chains (with relevant half lives) from thermal-neutron fission of U-235 are:

Kr-90 $\xrightarrow{33\ \text{s}}$ Rb-90 $\xrightarrow{2.7\ \text{min}}$ Sr-90 $\xrightarrow{28\ \text{yr}}$ Y-90 $\xrightarrow{64\ \text{hr}}$ Zr-90

Sr-90 is a serious health hazard, because it is bone-seeking.

Sb-131 $\xrightarrow{23\ \text{min}}$ Te-131 $\xrightarrow{24\ \text{min}}$ I-131 $\xrightarrow{8\ \text{days}}$ Xe-131

I-131 is a short-lived health hazard. It is thyroid-seeking.

I-135 $\xrightarrow{6.7\ \text{hr}}$ Xe-135 $\xrightarrow{9.2\ \text{hr}}$ Cs-135 (nearly stable) Fission yield $\gamma = 0.064$

Xe-135 is a strong absorber of thermal neutrons, with $\sigma_a = 3.5$ Mbarn .

Pm-149 $\xrightarrow{54\ \text{hr}}$ Sm-149 Fission yield $\gamma = 0.014$

Sm-149 is a strong absorber of thermal neutrons, with $\sigma_a = 53$ kbarn .

Se-87 $\xrightarrow{17\ \text{s}}$ Br-87 $\xrightarrow{55\ \text{s}}$ either {Kr-86 + n} or {Kr-87 $\xrightarrow{78\ \text{min}}$ Rb-87}

This chain leads to a “delayed neutron”.

Neutrons

Most neutrons are emitted within 10^{-13} s of fission, but some are only emitted when certain fission products, e.g. Br-87, decay.

The total yield of neutrons depends on the target and on the energy of the incident neutron. Some key values are:

Target nucleus	Fission induced by			
	Thermal neutron		Fast neutron	
	ν	η	ν	η
U-233	2.50	2.29	2.70	2.45
U-235	2.43	2.07	2.65	2.30
U-238	—	—	2.55	2.25
Pu-239	2.89	2.08	3.00	2.70

ν = number of neutrons emitted per fission

η = number of neutrons emitted per neutron absorbed

Delayed Neutrons

A reasonable approximation for thermal-neutron fission of U-235 is:

Precursor half life / s	55	22	5.6	2.1	0.45	0.15	Total
Mean life time of precursor ($1/\lambda_i$) / s	80	32	8.0	3.1	0.65	0.22	
Number of neutrons produced per 100 fission neutrons ($100\,\beta_i$)	0.03	0.18	0.22	0.23	0.07	0.02	0.75

Fission Energy

Kinetic energy of fission fragments	167 ± 5 MeV
Prompt γ-rays	6 ± 1 MeV
Kinetic energy of neutrons	5 MeV
Decay of fission products β	8 ± 1.5 MeV
γ	6 ± 1 MeV
Neutrinos (not recoverable)	12 ± 2.5 MeV
Total energy per fission	204 ± 7 MeV

Subtract neutrino energy and add neutron capture energy ⇒ ~200 MeV / fission

Nuclear Reactor Kinetics

Name	*Symbol*	*Concept*
Effective multiplication factor	k_{eff}	$\dfrac{\text{production}}{\text{removal}} = \dfrac{P}{R}$
Excess multiplication factor	k_{ex}	$\dfrac{P-R}{R} = k_{eff} - 1$
Reactivity	ρ	$\dfrac{P-R}{P} = \dfrac{k_{ex}}{k_{eff}}$
Lifetime	l	$\dfrac{1}{R}$
Reproduction time	Λ	$\dfrac{1}{P}$

Reactor Kinetics Equations

$$\frac{dn}{dt} = \frac{\rho - \beta}{\Lambda} n + \lambda c + s$$

$$\frac{dc}{dt} = \frac{\beta}{\Lambda} n - \lambda c$$

where n = neutron concentration

c = precursor concentration

β = delayed neutron precursor fraction $= \sum \beta_i$

λ = average precursor decay constant

Neutron Diffusion Equation

$$\frac{dn}{dt} = -\nabla.\underline{j} + (\eta - 1)\Sigma_a \phi + S$$

where $\underline{j} = -D\nabla\phi$ (Fick's Law)

$$D = \frac{1}{3\Sigma_s(1-\overline{\mu})}$$

with $\overline{\mu} =$ the mean cosine of the angle of scattering

Laplacian ∇^2

Slab geometry: $\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2} + \dfrac{\partial^2}{\partial z^2}$

Cylindrical geometry: $\dfrac{1}{r}\dfrac{\partial}{\partial r}\left(r\dfrac{\partial}{\partial r}\right) + \dfrac{1}{r^2}\dfrac{\partial^2}{\partial \theta^2} + \dfrac{\partial^2}{\partial z^2}$

Spherical geometry: $\dfrac{1}{r^2}\dfrac{\partial}{\partial r}\left(r^2\dfrac{\partial}{\partial r}\right) + \dfrac{1}{r^2 \sin\theta}\dfrac{\partial}{\partial \theta}\left(\sin\theta\dfrac{\partial}{\partial \theta}\right) + \dfrac{1}{r^2 \sin^2\theta}\dfrac{\partial^2}{\partial \psi^2}$

Bessel's Equation of 0th Order

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dR}{dr}\right) + R = 0$$

Solution is: $$R(r) = A_1 J_0(r) + A_2 Y_0(r)$$

$J_0(0) = 1$; $Y_0(0) = -\infty$;

The first zero of $J_0(r)$ is at $r = 2.405$.

$\int rJ_0(\beta r)dr = \frac{r}{\beta}J_1(\beta r)$ where $J_1(0) = 0$ and $J_1(2.405) = 0.5183$.

Diffusion and Slowing Down Properties of Moderators

Moderator	Density $\mathrm{g\,cm^{-3}}$	Σ_a $\mathrm{cm^{-1}}$	D cm	$L^2 = D/\Sigma_a$ $\mathrm{cm^2}$
Water	1.00	22×10^{-3}	0.17	$(2.76)^2$
Heavy Water	1.10	85×10^{-6}	0.85	$(100)^2$
Graphite	1.70	320×10^{-6}	0.94	$(54)^2$

In-core Fuel Management Equilibrium Cycle Length Ratio

For M-batch refueling: $$\theta = \frac{T_M}{T_1} = \frac{2}{M+1}$$

Enrichment of Isotopes

Value function: $$v(x) = (2x-1)\ln\left(\frac{x}{1-x}\right) \approx -\ln(x) \text{ for small } x$$

For any counter-current cascade at low enrichment:

Enrichment section reflux ratio: $$R_n \equiv \frac{L''_n}{P} = \frac{x_p - x'_{n+1}}{x'_{n+1} - x''_n}$$

Stripping section reflux ratio: $$R_n = \left[\frac{x_p - x_f}{x_f - x_w}\right]\left[\frac{x'_{n+1} - x_w}{x'_{n+1} - x''_n}\right]$$

Bateman's Equation

$$N_i = \lambda_1 \lambda_2 \ldots \lambda_{i-1} P \sum_{j=1}^{i} \frac{[1-\exp(-\lambda_j T)]\exp(-\lambda_j \tau)}{\lambda_j \prod_{\substack{k=1 \\ k \neq j}}^{i} (\lambda_k - \lambda_j)}$$

where N_i = number of atoms of nuclide i; T = filling time

λ_j = decay constant of nuclide j; τ = decay hold-up time after filling

P = parent nuclide production rate

Temperature Distribution

For axial coolant flow in a reactor with a chopped cosine power distribution, Ginn's equation for the non-dimensional temperature is:

$$\theta = \frac{T - T_{c1/2}}{T_{co} - T_{c1/2}} \sin\left(\frac{\pi L}{2L'}\right) = \sin\left(\frac{\pi x}{2L'}\right) + Q\cos\left(\frac{\pi x}{2L'}\right)$$

where L = fuel half-length

L' = flux half-length

$T_{c1/2}$ = coolant temperature at mid-channel

T_{co} = coolant temperature at channel exit

$Q = \dfrac{\pi \dot{m} c_p}{UA} \dfrac{L}{L'}$

with $\dot{m}$ = coolant mass flow rate

c_p = coolant specific heat capacity (assumed constant)

$A = 4\pi r_o L$ = surface area of fuel element

and for radial fuel geometry:

$$\frac{1}{U} = \underbrace{\frac{1}{h}}_{\text{bulk coolant}} + \underbrace{\frac{1}{h_s}}_{\text{scale}} + \underbrace{\frac{t_c}{\lambda_c}}_{\text{thin clad}} + \underbrace{\frac{r_o}{h_b r_i}}_{\text{bond}} + \underbrace{\frac{r_o}{2\lambda_f}\left(1 - \frac{r^2}{r_i^2}\right)}_{\text{fuel pellet}}$$

with h = heat transfer coefficient to bulk coolant

h_s = heat transfer coefficient of any scale on fuel cladding

t_c = fuel cladding thickness (assumed thin)

λ_c = fuel cladding thermal conductivity

r_o = fuel cladding outer radius

r_i = fuel cladding inner radius = fuel pellet radius

h_b = heat transfer coefficient of bond between fuel pellet and cladding

λ_f = fuel pellet thermal conductivity