Q1

(a) Mass defect $^{133}_{53}$I decay: $\Delta m = 132.90780 - 132.90591 = 0.00189$ u

$1\text{ u} \equiv 931.5\text{ MeV} \quad \therefore \quad E = 0.00189 \times 931.5 = 1.761\text{ MeV}$

Mass defect $^{133}_{54}$Xe decay: $\Delta m = 132.90591 - 132.90545 = 0.00046$ u

$1\text{ u} \equiv 931.5\text{ MeV} \quad \therefore \quad E = 0.00046 \times 931.5 = 0.428\text{ MeV}$

Assuming that the energy of the neutrino released in each decay is negligible, the energy released will be shared between the β-particle and the product nucleus. In practice the KE of the product nuclei will be small, so the KE of the β-particles will be close to the values calculated above. [10%]

(b)

(i) The first equation states that the rate of change of the $^{133}_{53}$I population is equal to the difference between the rate of production of $^{133}_{51}$Sb through fission (and therefore $^{133}_{53}$I soon thereafter) and the rate of loss of $^{133}_{53}$I by decay into $^{133}_{54}$Xe.

The second equation states the rate of change of the $^{133}_{54}$Xe population is equal to the difference between the rate of production of $^{133}_{54}$Xe by the decay of $^{133}_{53}$I and the rate of loss of $^{133}_{54}$Xe by decay into $^{133}_{55}$Cs.

These equations assume that the direct fission yields of $^{133}_{53}$I and $^{133}_{54}$Xe and the neutron capture cross-sections for both isotopes are negligible. [15%]

(ii) In equilibrium $\frac{dI}{dt} = 0$

$$\therefore \quad \gamma_{\mathrm{Sb}}\Sigma_{\mathrm{f}}\phi_0 - \lambda_{\mathrm{I}} I_0 = 0 \quad \Longrightarrow \quad I_0 = \frac{\gamma_{\mathrm{Sb}}\Sigma_{\mathrm{f}}\phi_0}{\lambda_{\mathrm{I}}}$$

and $\frac{dX}{dt} = 0$ $\quad \therefore \quad \lambda_{\mathrm{I}} I_0 - \lambda_{\mathrm{X}} X_0 = 0 \quad \Longrightarrow \quad \lambda_{\mathrm{I}} I_0 = \lambda_{\mathrm{X}} X_0$

$$\therefore \quad X_0 = \frac{\lambda_{\mathrm{I}} I_0}{\lambda_{\mathrm{X}}} = \frac{\gamma_{\mathrm{Sb}}\Sigma_{\mathrm{f}}\phi_0}{\lambda_{\mathrm{X}}}$$

[10%]

(c) Consider first the time variation of the $^{133}_{53}$I population.

$$\frac{dI}{dt} = -\lambda_{\mathrm{I}} I$$

Given that $I = I_0$ at $t = 0$, by inspection this has a solution

$$I = I_0 \mathrm{e}^{-\lambda_{\mathrm{I}} t}$$

Now consider the time variation of the $^{133}_{54}$Xe population.

$$\frac{dX}{dt} = \lambda_{\mathrm{I}} I - \lambda_{\mathrm{X}} X$$

Substituting for I and re-arranging:

$$\frac{dX}{dt} + \lambda_{\mathrm{X}} X = \lambda_{\mathrm{I}} I_0 \mathrm{e}^{-\lambda_{\mathrm{I}} t} \qquad (1.1)$$

By inspection, the complementary function will be:

$$X_{\mathrm{cf}} = C\mathrm{e}^{-\lambda_{\mathrm{X}} t}$$

where C is a constant the value of which depends on the initial conditions.

Also, by inspection, the particular integral will be of the form:

$$X_{\text{pi}} = D\text{e}^{-\lambda_{\text{I}}t}$$

To find the value of constant D substitute in equation (1.1):

$$-\lambda_{\text{I}}D\text{e}^{-\lambda_{\text{I}}t} + \lambda_{\text{X}}D\text{e}^{-\lambda_{\text{I}}t} = \lambda_{\text{I}}I_0\text{e}^{-\lambda_{\text{I}}t} \quad \Longrightarrow \quad D = \frac{\lambda_{\text{I}}I_0}{\lambda_{\text{X}} - \lambda_{\text{I}}}$$

Thus, the general solution for X is:

$$X = X_{\text{cf}} + X_{\text{pi}} = C\text{e}^{-\lambda_{\text{X}}t} + \frac{\lambda_{\text{I}}I_0}{\lambda_{\text{X}} - \lambda_{\text{I}}}\text{e}^{-\lambda_{\text{I}}t} \qquad (1.2)$$

If the initial condition is that $X = 0$ at $t = 0$, then equation (1.2) gives

$$0 = C + \frac{\lambda_{\text{I}}I_0}{\lambda_{\text{X}} - \lambda_{\text{I}}} \quad \Longrightarrow \quad C = -\frac{\lambda_{\text{I}}I_0}{\lambda_{\text{X}} - \lambda_{\text{I}}}$$

$$\therefore \quad X = -\frac{\lambda_{\text{I}}I_0}{\lambda_{\text{X}} - \lambda_{\text{I}}}\text{e}^{-\lambda_{\text{X}}t} + \frac{\lambda_{\text{I}}I_0}{\lambda_{\text{X}} - \lambda_{\text{I}}}\text{e}^{-\lambda_{\text{I}}t} = \frac{\lambda_{\text{I}}I_0}{\lambda_{\text{X}} - \lambda_{\text{I}}}\left[\text{e}^{-\lambda_{\text{I}}t} - \text{e}^{-\lambda_{\text{X}}t}\right]$$

[40%]

(d) The $^{133}_{54}$Xe activity will peak when X is a maximum. This occurs when $\frac{dX}{dt} = 0$.

Differentiating the result in part (c):

$$\therefore \quad \frac{dX}{dt} = \frac{\lambda_{\text{I}}I_0}{\lambda_{\text{X}} - \lambda_{\text{I}}}\left[-\lambda_{\text{I}}\text{e}^{-\lambda_{\text{I}}t} + \lambda_{\text{X}}\text{e}^{-\lambda_{\text{X}}t}\right]$$

$$\therefore \quad -\lambda_{\text{I}}\text{e}^{-\lambda_{\text{I}}t} + \lambda_{\text{X}}\text{e}^{-\lambda_{\text{X}}t} = 0$$

$$\therefore \quad \text{e}^{(\lambda_{\text{I}} - \lambda_{\text{X}})t} = \frac{\lambda_{\text{I}}}{\lambda_{\text{X}}}$$

$$\therefore \quad t_m = \frac{1}{\lambda_{\text{I}} - \lambda_{\text{X}}}\ln\left(\frac{\lambda_{\text{I}}}{\lambda_{\text{X}}}\right)$$

$$\lambda = \frac{\ln(2)}{T_{1/2}}$$

$$\therefore \quad \lambda_{\text{I}} = \frac{\ln(2)}{20.8 \times 86400} = 3.857 \times 10^{-7}\ \text{s}^{-1}$$

$$\therefore \quad \lambda_{\text{X}} = \frac{\ln(2)}{5.24 \times 86400} = 1.531 \times 10^{-6}\ \text{s}^{-1}$$

$$\therefore \quad t_m = \frac{1}{3.857 \times 10^{-7} - 1.531 \times 10^{-6}}\ln\left(\frac{3.857 \times 10^{-7}}{1.531 \times 10^{-6}}\right) = 1.204 \times 10^{6}\ \text{s or } 13.9\ \text{days}$$

In this case, because $\lambda_{\text{X}} \approx 4\lambda_{\text{I}}$, secular equilibrium is approached in which the activities of the $^{133}_{53}$I and $^{133}_{54}$Xe are approximately equal. However, as (from part (a)) the β-particle emitted by $^{133}_{54}$Xe has only about a quarter of the energy of the β-particle emitted by $^{133}_{53}$I, this effect will 'only' increase the dose received by someone ingesting any $^{133}_{53}$I by about 25% above that due to the $^{133}_{53}$I alone. [25%]

Assessor's Comments

All candidates: 102 attempts, Average raw mark 15.0/20, Maximum 20, Minimum 6.

An extremely popular question attempted by all but one candidate and very done well by many.

Several candidates lost marks due to failing to state assumptions.
Many candidates incorrectly 'double counted' for the mass of the beta particle in calculating the energy released in the decay reactions in part (a) – it is implicitly accounted for in the atomic mass of the decay product.
A surprising number of candidates referred to the "fission of Sb". Sb-133 is produced as a product of the fission of U-235.
A number of candidates confused Xe-133 with Xe-135, or assumed incorrectly they would have similar microscopic cross-sections.
Some candidates struggled to solve the differential equations in part (c) despite their similarities to the familiar xenon poisoning equations. A number applied the wrong boundary conditions, even though they are clearly stated in the question. Others confused the particular integral and complementary function.
Many answers to part (d) made rather generic comments rather than, as was expected, taking note of specifics relating to the half-lives and beta energies of the isotopes in question.
Many answers to part (d) failed to distinguish adequately between dose rate and accumulated dose.

Q2

(a) The values of α and β are determined by the boundary conditions for the flux solution. If extrapolation distances can be neglected, these are that $\phi = 0$ at $r = R$ and at $z = \pm H/2$.

The first of these implies that $J_0(\alpha R) = 0$

$$\therefore \quad \alpha R = 2.405 \quad \Longrightarrow \quad \alpha = \frac{2.405}{R}$$

The second implies that $\cos(\beta H/2) = 0$

$$\therefore \quad \frac{\beta H}{2} = \frac{\pi}{2} \quad \Longrightarrow \quad \beta = \frac{\pi}{H}$$

[15%]

(b) The axial form factor for a cylindrical geometry reactor is defined as

$$F_Z = \frac{\text{the flux at the centre of the core}}{\text{the average flux along the central channel}}$$

$$\therefore \quad F_Z = \frac{\phi_0}{\frac{1}{H}\int_{-H/2}^{H/2} \phi_0 \cos\left(\frac{\pi}{H} z\right) dz} = \frac{\pi H}{2H \sin\left(\frac{\pi}{H}\frac{H}{2}\right)} = \frac{\pi}{2} = 1.571$$

[10%]

(c)

(i)

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \phi}{\partial r}\right) + \frac{\partial^2 \phi}{\partial z^2} - \frac{1}{M^2}\phi = 0$$

Assume that $\phi(r, z) = F(r)Z(z)$

$$\therefore \quad \frac{Z}{r}\frac{\partial}{\partial r}\left(r\frac{\partial F}{\partial r}\right) + F\frac{\partial^2 Z}{\partial z^2} - \frac{1}{M^2}FZ = 0$$

Dividing through by FZ

$$\therefore \quad \frac{1}{Fr}\frac{\partial}{\partial r}\left(r\frac{\partial F}{\partial r}\right) + \frac{1}{Z}\frac{\partial^2 Z}{\partial z^2} - \frac{1}{M^2} = 0$$

This implies that

$$\frac{1}{Fr}\frac{d}{dr}\left(r\frac{dF}{dr}\right)+\alpha^2=0$$

$$\frac{1}{Z}\frac{d^2Z}{dz^2}-\gamma^2=0$$

with $$\alpha^2-\gamma^2=-\frac{1}{M^2}$$

The minus sign in the equation in z is needed because of the minus sign before the $1/M^2$ term in these equations.

The general solution for the equation in r is

$$F(r)=PJ_0(\alpha r)+QY_0(\alpha r)$$

where P and Q are arbitrary constants and J_0 and Y_0 are ordinary Bessel functions of zero order.

As $r\to 0$, $Y_0(\alpha r)\to-\infty$. This would give infinite flux on the reflector centreline, which is physically impossible, so $Q=0$.

The boundary condition that $\phi=0$ at $r=R$ implies that, as in (a),

$$J_0(\alpha R)=0 \quad\Rightarrow\quad \alpha R=2.405 \quad\Longrightarrow\quad \alpha=\frac{2.405}{R}$$

The general solution for the equation in z is

$$Z(z)=A\exp(\gamma z)+C\exp(-\gamma z)$$

Combining these results, the general solution for the flux distribution in the reflector is

$$\phi(r,z)=[A\exp(\gamma z)+C\exp(-\gamma z)]\,J_0(\alpha r)$$ [30%]

(ii) As established above

$$\alpha^2-\gamma^2=-\frac{1}{M^2}$$

[5%]

(d)

(i) In the core $$\phi(r,z)=\phi_0 J_0(\alpha r)\cos(\beta z)$$

In the reflector $$\phi(r,z)=\phi_1 J_0(\alpha r)\exp(-\gamma z)$$

Flux continuity at the core-reflector interface ($z=H/2$) gives

$$\phi_0\cos(\beta H/2)=\phi_1\exp(-\gamma H/2)$$

$$\therefore\quad \phi_1=\phi_0\frac{\cos(\beta H/2)}{\exp(-\gamma H/2)}$$

[10%]

(ii) In the core

$$\frac{\partial\phi}{\partial z}=-\beta\phi_0 J_0(\alpha r)\sin(\beta z)$$

And in the reflector

$$\frac{\partial\phi}{\partial z}=-\gamma\phi_1 J_0(\alpha r)\exp(-\gamma z)$$

So, current continuity at the core-reflector interface ($z=H/2$) gives

$$D_c\beta\phi_0\sin(\beta H/2)=D_r\gamma\phi_1\exp(-\gamma H/2)$$

From (i), $\phi_0 \cos(\beta H/2) = \phi_1 \exp(-\gamma H/2)$

$$\therefore \quad D_c \beta \phi_0 \sin(\beta H/2) = D_r \gamma \phi_0 \cos(\beta H/2)$$

$$\therefore \quad \gamma = \frac{D_c}{D_r} \beta \tan(\beta H/2)$$

[15%]

(e) With axial reflectors the height of the active core is now $0.8H$. To a good approximation it can be assumed that the flux distribution over the active core is the same as for the original bare (unreflected) reactor, i.e. β is the same.

$$\therefore \quad F_Z = \frac{\phi_0}{\frac{1}{0.8H}\int_{-0.4H}^{0.4H} \phi_0 \cos\left(\frac{\pi}{H} z\right) dz} = \frac{\pi H}{2.5H \sin\left(\frac{\pi}{H}\frac{2H}{5}\right)} = \frac{\pi}{2.378} = 1.321$$

The axial form factor is much improved, as expected. Reflectors are an effective way of reducing form factors. [15%]

Assessor's Comments

All candidates: 96 attempts, Average raw mark 13.3/20, Maximum 20, Minimum 1.

A very popular question attempted by 93% of candidates and done well by many.
A number of candidates wasted time deriving the result for $\phi(r, z)$ given in the preamble.
Some candidates could not recall the definition of axial form factor or defined it correctly but then set up an expression inconsistent with the definition.
Even with a correct expression, several candidates were unable to correctly calculate the average axial flux.
Many answers to part (c) lacked rigour. Candidates had issues with the signs of the α^2 and γ^2 terms. There were some claims that an SHM equation yielded solutions of the form $\exp(\pm\gamma z)$.
In tackling part (d), several candidates failed to realise that $\beta H \neq \pi$ for a reactor with axial reflectors.
Form factor calculations in part (e) sometimes went astray because candidates did not properly adjust their expression to account for the new length of the channel, or used the flux distribution in the reflector in the calculation.
Some calculations of form factors less than unity passed without comment.

Q3

(a) CANDU reactors are refuelled on-line. As the fuel is natural uranium the reactivity change in replacing a burnt assembly with a fresh one is comparatively small, and the reactor is naturally kept close to critical.

PWRs (typical fuel enrichment 3–4%) are refuelled off-line at 12-, 18- or even 24-month intervals.

The procedure requires the reactor to be depressurised so that the top of the pressure vessel can be removed, allowing fuel assemblies to be removed or shuffled about. Refuelling takes between one and two months. The more often a reactor is refuelled, the less electricity it will generate per annum. Thus, some sort of compromise must be struck between the conflicting goals of achieving high fuel burnup and high reactor availability. In practice, most PWRs are operated using a 3- or 4-batch strategy.

For reactors, like CANDU, that are refuelled on-line there is no real scope for in-core fuel management, but for reactors that are refuelled off-line the design of the core loading pattern (LP) is an important issue. The purpose of in-core fuel management is to control local spatial

effects resulting from the actual physical arrangement of the fuel. For example, fresh fuel will tend to amplify the local flux distribution above the overall 'cosinusoidal' shape, while fuel with a large accumulated burnup will tend to depress it. Thus, if too many fresh fuel elements are clustered together, there is likely to be a local power peak (a hot spot).

The LP designer can have several (conflicting) objectives to meet:

- to limit the magnitude of any local power peaking;
- to limit the amount of radial neutron leakage from the core so as to reduce the irradiation damage to the pressure vessel;
- to maximise the average burnup accumulated by the fuel; or
- to maximise the overall reactivity of the core so as to reduce the amount of fresh fuel required.

To strike an appropriate balance between these competing objectives some sort of 'checkerboarding' is usually used. It is also common to load *burnable poisons* with fresh fuel assemblies to overcome power peaking problems in otherwise desirable core designs. [40%]

(b)

(i) Using the linear reactivity model

$$\rho = \rho_0 \left[1 - \frac{\tau}{T_1}\right]$$

where ρ_0 is the initial reactivity (which depends on the batch enrichment), τ is the burnup and T_1 is the one-batch cycle length (in units of burnup). As the reactor power is constant, burnup is proportional to time, so work in units of time.

From page 7 of the 4M16 data sheet, the equilibrium cycle length in 3-batch operation

$$T_M = \frac{2}{M+1} T_1 \quad \text{with } M = 3 \implies T_1 = 2T_3 = 24 \text{ months}$$

Hence, in 2-batch operation,

$$T_M = \frac{2}{M+1} T_1 \quad \text{with } M = 2 \implies T_2 = \frac{2}{3} T_1 = \frac{2}{3} \times 24 = 16 \text{ months}$$

The principal disadvantage of 2-batch operation (compared to 3-batch) is that fuel utilisation (discharge burnup) is reduced:

$$B_M = MT_M = \frac{2M}{M+1} T_1 \implies \frac{B_2}{B_3} = \frac{2 \times 2}{2+1} \cdot \frac{3+1}{2 \times 3} = \frac{8}{9}$$

A second disadvantage is that it will be harder to schedule refuelling outages at times of lower electricity demand. [20%]

(ii) At the start of the first cycle of 2-batch operation, the reactor inventory will be:

1 batch (50% of the reactor) of fresh fuel

1 batch (1/3rd of the reactor) of once-burnt (on the three-batch cycle) fuel

1 batch (1/6th of the reactor) of twice-burnt (on the three-batch cycle) fuel

The oldest of the 3-batch scheme batches (1/3rd of the reactor, thrice-burnt) will have been completely discharged at the end of the previous cycle and $\frac{1}{2} - \frac{1}{3} = \frac{1}{6}$ (i.e. half) of the second oldest (twice-burnt) batch will also be discharged to make space for the increased batch size.

Let ψ_1 be the length of the first 2-batch cycle. The end-of-cycle (EOC) condition will be

$$\frac{1}{2}\rho_0\left[1-\frac{\psi_1}{T_1}\right]+\frac{1}{3}\rho_0\left[1-\frac{T_3+\psi_1}{T_1}\right]+\frac{1}{6}\rho_0\left[1-\frac{2T_3+\psi_1}{T_1}\right]=0$$

freshest batch once-burnt batch twice-burnt batch

$$\therefore\quad 1-\frac{\psi_1}{2T_1}-\frac{T_3+\psi_1}{3T_1}-\frac{2T_3+\psi_1}{6T_1}=0$$

$$\therefore\quad 6T_1-3\psi_1-2T_3-2\psi_1-2T_3-\psi_1=0$$

$$\therefore\quad 6\psi_1=6T_1-4T_3$$

$$\therefore\quad \psi_1=T_1-\frac{2}{3}T_3$$

From (b)(i) $T_1=2T_3$

$$\therefore\quad \psi_1=2T_3-\frac{2}{3}T_3=\frac{4}{3}T_3=\frac{4}{3}\times 12=16\text{ months}$$

In subsequent cycles there is true two-batch operation but (potentially) with non-equilibrium cycle lengths. With successive cycle lengths ψ_n and ψ_{n-1}, the EOC condition is

$$\frac{1}{2}\rho_0\left[1-\frac{\psi_n}{T_1}\right]+\frac{1}{2}\rho_0\left[1-\frac{\psi_n+\psi_{n-1}}{T_1}\right]=0$$

freshest batch oldest batch

$$\therefore\quad 1-\frac{\psi_n}{2T_1}-\frac{\psi_n+\psi_{n-1}}{2T_1}=0$$

$$\therefore\quad T_1-\frac{\psi_n}{2}-\frac{\psi_n}{2}-\frac{\psi_{n-1}}{2}=0$$

$$\therefore\quad \psi_n=T_1-\frac{\psi_{n-1}}{2}$$

Using this recurrence relation

$$\therefore\quad \psi_2=T_1-\frac{\psi_1}{2}=24-\frac{16}{2}=16\text{ months}$$

Similarly, $\psi_3=\psi_4=16$ months, i.e. equilibrium operation is established immediately! [40%]

Assessor's Comments

All candidates: 59 attempts, Average raw mark 11.5/20, Maximum 20, Minimum 4.

The third most popular question, attempted by 57% of candidates.
Many candidates only partially answered part (a). Some confused CANDU reactors with AGRs.
Some candidates confused in-core/out-of-core fuel management with online and offline refuelling.
There were several instances of the end-of-cycle condition for the first transition cycle in part (c) being correctly set up but incorrectly solved; in more than one case because apparently the candidate couldn't read their own handwriting.
There were a few brave attempts to solve part (c) graphically, none of which properly accounted for the different batch sizes in the first transition cycle.
Many candidates made extra work for themselves by not recognising that a recurrence relation could be used to find the second and subsequent transition cycle lengths.

Q4

(a) In order to maintain criticality, it is necessary to enrich the fuel used in most designs of civil nuclear power plant. Neutrons are absorbed in the moderator and the fuel cladding as well as being lost to non-fission reactions and lost from the reactor altogether (leakage). Light water (the coolant/moderator used in PWRs) has a higher neutron capture cross-section than the other two moderators commonly used (graphite and heavy water), and the zircaloy cladding is also more neutron absorbing than alternatives like magnesium or aluminium. To compensate for the greater absorption of neutrons in coolant/moderator and cladding, the fuel must be enriched. [10%]

(b) Reprocessing was essential for the old style Magnox fuel because it is not suitable for long-term storage. PWR and AGR fuel can be stored safely, but there are still political problems finding suitable sites. The decisions by the UK and France to reprocess the more modern fuels were taken at a time of great energy shortage and the plan, at that time, was to recover the Pu to fuel fast reactors and also to use the recovered U in breeding blankets. The only real use for the recovered Pu at present is in the form of mixed oxide fuel (MOX). With increasing energy prices reprocessing could again become attractive.

The main downside of reprocessing is that, although it separates the various wastes, it does result in an overall increase in waste volumes and there are some discharges to the environment. There is also the associated proliferation risk and the question of what to do with the recovered Pu. In practice, the Pu is of little use for weapons as it contains too much Pu-240 and Pu-241 whereas weapons grade material is mostly Pu-239. [20%]

(c) Open cycle mass balance:

$$F = P + W \quad \Rightarrow \quad F = 500 + W \qquad (4.1)$$

$$x_F F = x_P P + x_W W \quad \Longrightarrow \quad 0.007F = 0.035 \times 500 + 0.003W$$

$$\therefore \quad 0.007F = 17.5 + 0.003W$$

$0.003 \times$ (4.1) $$0.003F = 1.5 + 0.003W$$

$$\therefore \quad 0.004F = 16.0 \quad \Rightarrow \quad F = 4000 \text{ tonnes}$$

$$\therefore \quad W = F - 500 = 3500 \text{ tonnes}$$

$$SWU = P(-\ln(x_P)) + W(-\ln(x_W)) - F(-\ln(x_F))$$

$$\therefore \quad SWU = 500(-\ln(0.035)) + 3500(-\ln(0.003)) - 4000(-\ln(0.007))$$

$$\therefore \quad SWU = 1676.2 + 20332.0 - 19847.4 = 2160.8 \text{ tonnes SWU}$$ [15%]

(d) Closed cycle:

$$\text{Recovered uranium } R = 0.96F = 0.96 \times 500 = 480 \text{ tonnes}$$

$$\text{Reprocessing losses (1\%)} = 0.01R = 4.8 \text{ tonnes}$$

$$\therefore \quad \text{Available uranium } F_r = 0.99R = 475.2 \text{ tonnes}$$

Mass balance on recovered uranium:

$$F_r = P_r + W_r \quad \Rightarrow \quad 475.2 = P_r + W_r \qquad (4.2)$$

$$x_r F_r = x_P P_r + x_W W_r \quad \Longrightarrow \quad 0.008 \times 475.2 = 3.8016 = 0.035P_r + 0.003W_r$$

$0.003 \times$ (4.2) $$0.003 \times 475.2 = 1.4256 = 0.003P_r + 0.003W_r$$

$$\therefore \quad 3.8016 - 1.4256 = 2.376 = 0.032P_r \quad \Rightarrow \quad P_r = 74.25 \text{ tonnes}$$

$$\therefore \quad W_r = F_r - P_r = 475.2 - 74.25 = 400.95 \text{ tonnes}$$

Product required from fresh feed $P' = P - P_r = 500 - 74.25 = 425.75$ tonnes

The amount of fresh feed now required can be found by scaling the result in (c):

$$F' = F \times \frac{P'}{P} = 4000 \times \frac{425.75}{500} = 3406 \text{ tonnes}$$

$\therefore$ Available feed saving per year $= F - F' = 4000 - 3406 = 594$ tonnes

For the reprocessed fuel:

$$SWU_r = P_r(-\ln(x_P)) + W_r(-\ln(x_W)) - F_r(-\ln(x_r))$$

$$\therefore \quad SWU_r = 74.25(-\ln(0.035)) + 400.95(-\ln(0.003)) - 475.2(-\ln(0.008))$$

$$\therefore \quad SWU_r = 248.92 + 2329.18 - 2294.41 = 283.69 \text{ tonnes SWU}$$

The amount of SWU for enriching the fresh feed now required can be found by scaling the result in (c):

$$SWU' = SWU \times \frac{P'}{P} = 2160.8 \times \frac{425.75}{500} = 1839.92 \text{ tonnes SWU}$$

So, the total SWU with reprocessing

$$SWU_R = SWU' + SWU_r = 1839.92 + 283.69 = 2123.6 \text{ tonnes SWU}$$

$\therefore$ Available SWU saving per year $= SWU - SWU_R = 2160.8 - 2123.6 = 37.2$ tonnes SWU [35%]

(e) *Open cycle costs*:

Feed cost: $C_F = c_F F = 100 \times 4000 \times 10^3 = \$400 \times 10^6$

SWU cost: $C_{SWU} = c_{SWU} SWU = 100 \times 2160.8 \times 10^3 = \$216.08 \times 10^6$

Disposal cost: $C_D = c_D P = 1000 \times 500 \times 10^3 = \$500 \times 10^6$

$\therefore$ Total cost: $C = C_F + C_{SWU} + C_D = \$1116.8 \times 10^6$

Closed cycle costs:

Feed cost: $C_F = c_F F' = 100 \times 3406 \times 10^3 = \$340.6 \times 10^6$

SWU cost: $C_{SWU} = c_{SWU} SWU_R = 100 \times 2123.6 \times 10^3 = \$212.36 \times 10^6$

Reprocessing cost: $C_R = c_R P = 3000 \times 500 \times 10^3 = \$1500 \times 10^6$

$\therefore$ Total cost: $C = C_F + C_{SWU} + C_R = \$2053.0 \times 10^6$

Thus, the closed cycle costs significantly more than the open cycle. It would take a large (more than ten-fold) increase in uranium costs to make reprocessing economically viable. [20%]

Assessor's Comments

All candidates: 51 attempts, Average raw mark 11.9/20, Maximum 20, Minimum 0.

The least popular question, attempted by just under half the candidates.
In answering part (a) most answers mentioned the high capture cross-section of the light water coolant/moderator; few mentioned the cladding.
Many part (b) answers focused on the general advantages/disadvantages of reprocessing rather than historical/present-day issues.
A few part (b) answers confused enrichment with reprocessing.
In answering parts (c) and (d) a few candidates were unable to do the basic mass balances required.

Several part (d) answers failed to account for the losses in reprocessing.
Some candidates seemed to think that it was the waste (tails) stream from the enrichment plant being reprocessed, not the original fuel (product) after passing through the reactor.
Some answers to part (d) neglected to account for the SWU associated with enriching the reprocessed fuel.
Some of the part (e) cost analyses failed to note that some cost elements were based on reactor feed mass. Others paid insufficient attention to detail in analysing the processes.
Some cost calculations with incorrect inputs were set out so badly that it was impossible to assign partial credit.