EGT3
ENGINEERING TRIPOS PART IIB

Thursday 30 April 2026 14.00 to 15.40

Module 4D5

FOUNDATION ENGINEERING

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 4D5 Foundation Engineering Databook (21 pages)
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

1 An offshore wind turbine is to be installed at a site where the water depth is 20 m and the ground conditions comprise normally consolidated clay with an undrained strength increasing linearly with depth from zero at the mudline at a rate of 3 kPa m^{-1}. The unit weight of the clay can be assumed to be 16 kN m^{-3} at all depths. The wind turbine exerts design loads on the foundation of 6 MN vertically and 4 MN horizontally at a height of 60 m above sea-level.

(a) Explain three reasons why pile foundations might be preferred to a gravity base in this scenario. [30%]

(b) The foundation is chosen to be a 5 m diameter tubular pile constructed from steel with a design strength of 400 MPa. By considering only the horizontal load and its associated moment, what length of pile and what wall thickness are required to carry the load? [40%]

(c) What is the vertical capacity of the pile with dimensions as chosen in part (b)? [20%]

(d) What other considerations might need to be taken into account in the design of the foundation? [10%]

2 (a) Derive from first principles an equation for the shaft stiffness of a rigid pile with radius R and length L in a uniform elastic soil with a shear stiffness G. [20%]

(b) A 2×2 pile group consists of four piles each of diameter 1 m and length 20 m with a pile spacing of 8 m, founded in soil with a uniform shear stiffness of 10 MPa, as shown in Fig. 1. Calculate:

(i) the shaft stiffness of an individual pile within the group assuming the pile material to be rigid; [10%]

(ii) the stiffness of the four pile group assuming the pile cap does not rest on the soil. [30%]

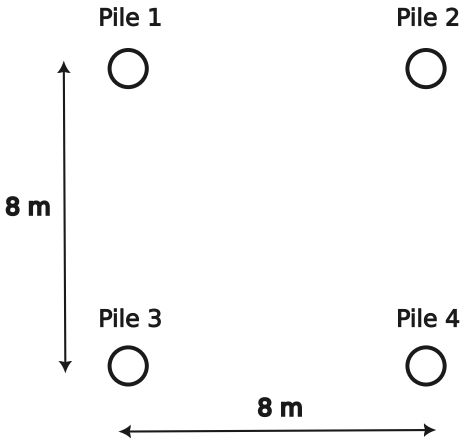


Fig. 1

Pile number	Ground settlement (mm)
1	2
2	4
3	4
4	10

Table 1

(c) Construction of a nearby tunnel causes ground movements as shown in Table 1. If the total load on the pile group is 10 MN:

(i) assuming the pile heads to be pinned to the cap, show that diagonally opposite piles in the group must each carry the same load; [10%]

(ii) calculate the average settlement and rotation of the pile group due to the tunneling-induced ground displacements and the load carried by each pile in the group. [30%]

3 (a) Describe the main types of embedded retaining walls, outlining their typical construction methods and key features. [15%]

(b) Figure 2 shows an excavation with a depth h = 5 m, supported by a steel sheet pile wall with an embedded depth d = 3.2 m and one level of temporary props at the top. The soil profile consists of a 6 m layer of sand with voids ratio $e = 0.9$, specific gravity $G_\mathrm{S} = 2.65$, and characteristic friction angle $\varphi'_k = 33°$ followed by a second layer of denser sand ($e = 0.7$, $G_\mathrm{S} = 2.7$, $\varphi'_k = 39°$) extending to significant depth. The groundwater level is 3 m below ground surface.

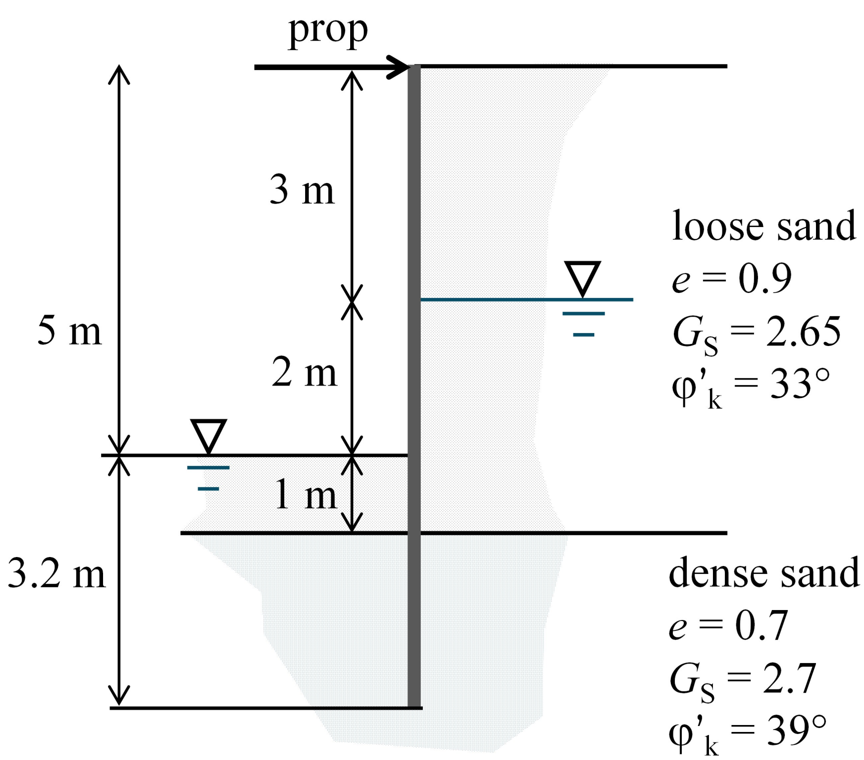


Fig. 2

(i) Compute the unit weight of the two layers of sand in dry and saturated conditions. [5%]

(ii) Assuming that the mobilised friction at the interface between the wall and the soil on the passive side is $\delta = \varphi'/3$, compute the design active and passive lateral earth pressure using $\gamma_{\varphi'} = 1.25$ and Rankine's and Lancellotta's static solutions, respectively:

$$K_\mathrm{A} = \frac{1 - \sin\varphi'}{1 + \sin\varphi'}$$

$$K_{\mathrm{P}} = \frac{\cos\delta}{1-\sin\varphi'}\left[\cos\delta + \sqrt{(\sin\varphi')^2 - (\sin\delta)^2}\right] e^{2\Theta\tan\varphi'}$$

where:

$$2\Theta = \sin^{-1}\left(\frac{\sin\delta}{\sin\varphi'}\right) + \delta$$

[10%]

(iii) Check that the design restoring moment is larger than the design driving moment: $M_R, d \geq M_D, d$. You can assume a constant gradient for the hydraulic head loss. [35%]

(iv) Check if the prop is still needed, should the excavation be carried out in submerged conditions. [35%]

4 (a) Describe the possible methods of tunnel construction in clay, outlining the main characteristics and typical applications of each method. [20%]

(b) A tunnel of diameter $D = 6$ m is to be constructed with its axis at a depth $z = 15$ m below ground level in a stiff clay. The unit weight of the clay is $\gamma = 20\,\mathrm{kN\,m^{-3}}$, the undrained shear strength of the clay is $s_u = 100$ kPa (constant with depth), the coefficient of earth pressure at rest is $k_0 = 1$, and the elastic shear modulus is $G = 30$ MPa.

(i) The values of the coefficient of earth pressure at rest, the undrained shear strength, and the shear modulus were obtained from a self-boring pressuremeter test undertaken at the level of the tunnel axis. Describe, with appropriate sketches, how these parameters are determined from the test data, giving two methods by which the undrained shear strength can be estimated. [25%]

(ii) Can tunnel construction be carried out without providing support? Justify your answer making use of the chart in Fig. 3. [15%]

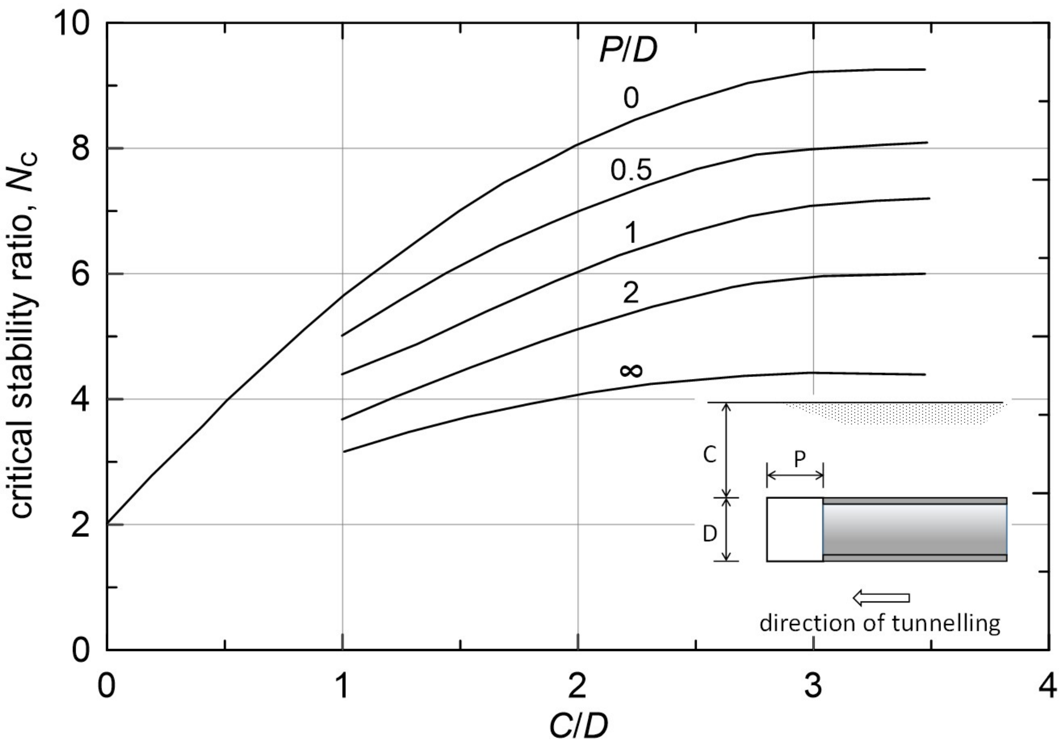


Fig. 3

(iii) A pipeline runs with its axis perpendicular to that of the tunnel at a depth of 3 m below ground level. Assuming that the tunnel is temporarily unlined, and that the pipeline is of small diameter and flexible, therefore deforming with the ground, estimate the maximum settlement of the pipeline. [40%]

END OF PAPER

Cambridge University Engineering Department

Supplementary Databook

Module 4D5: Foundation Engineering

SKH. January 2024

Section 1: Empirical correlations for geotechnical data

1.1 Undrained shear strength of clays (s_u)

$\left(\frac{s_u}{\sigma'_v}\right)_{nc} \approx 0.11 + 0.37\, I_p$ for normally consolidated clay, where I_p is the plasticity index

$\frac{s_u}{\sigma'_v} \approx \left(\frac{s_u}{\sigma'_v}\right)_{nc} n^{\Lambda}$ where $n = \sigma'_{v,c} / \sigma'_v$ is overconsolidation ratio; $\Lambda \approx 0.8$

$s_u = \frac{(q_{penetrometer} - \sigma_v)}{N_{penetrometer}}$ from q at tip load cell, where $N_{cone} \approx 14 \pm 2$; $N_{T\text{-}bar} \approx 12 \pm 2$

$s_u \approx 4.5 N_{60}$ kPa from SPT blow-count N_{60} in Standard Penetration Test

1.2 Drained shear strength of sands (friction and dilatancy)

definition of relative dilatancy	$I_R = I_D I_C - 1$
definition of relative density	$I_D = (e_{max} - e)(e_{max} - e_{min})$
SPT blow-count correlation	$I_D \approx [N_{60} / (20 + 0.2\, \sigma'_v \text{ kPa})]^{0.5}$
definition of relative crushability	$I_C = \ln(\sigma_c / p')$
aggregate crushing stress	$\sigma_c \approx$ shelly carbonate sand 5 000 kPa
	quartz sand 20 000 kPa
	quartz silt 80 000 kPa
CPT correlation (q_{cone} , σ'_v in kPa)	$I_D \approx 0.27(\ln q_{cone} - 0.5 \ln \sigma'_v) - 1.29 \pm 0.15$ (higher, σ_c lower)
peak friction correlation	$(\phi_{max} - \phi_{crit}) \approx 0.8\ \psi_{max} \approx 5° \times I_R$ in plane strain
	$(\phi_{max} - \phi_{crit}) \approx 3° \times I_R$ in axisymmetric strain
peak dilatancy rate	$(-\delta\varepsilon_v / \delta\varepsilon_1)_{max} \approx 0.3 \times I_R$ in all conditions
critical state friction angle	$\phi_{crit} \approx 32°$ (uniform, rounded) $\rightarrow$ 40° (well-graded, angular)

1.3 Stiffness of clays

initial linear elastic shear stiffness $G_0 = \frac{B}{(1+e)^{2.4}}(p')^{0.5}$ with G_0, p' in kPa

$B \approx 20\,000$

normalised secant shear stiffness $\frac{G}{G_0} \approx \frac{1}{1+\left(\frac{\gamma}{\gamma_{ref}}\right)^a}$

hyperbolic curvature parameter $a \approx 0.68$ for $n < 1.5$, $a \approx 0.77$ for $n > 1.5$

reference shear strain $\gamma_{ref} \approx A\, w_L 10^{-3}$ e.g. writing liquid limit 40% as $w_L = 0.4$

$A \approx 1.35$ for $n < 1.5$; $A \approx 1.02$ for $n > 1.5$

mobilised shear strength $\frac{\tau_{mob}}{s_u} \approx \left(\frac{\gamma}{\gamma_u}\right)^{0.5}$ for $s_{mob} < s_u$

$\gamma_u \approx 0.01$ to 0.04 increasing with plasticity index I_p

1.4 Stiffness of sands

initial linear elastic shear stiffness $G_0 = \frac{B}{(1+e)^3}(p')^{0.5}$ with G_0, p' in kPa

$B \approx 57\,600$

normalised secant shear stiffness $\frac{G}{G_0} = \frac{1}{1+\left(\frac{\gamma-\gamma_e}{\gamma_{ref}}\right)^a}$

hyperbolic curvature parameter $a = U_c^{-0.075}$ e.g. $a = 0.9$ at uniformity coefficient $U_c = 4$

reference shear strain $\gamma_{ref} = U_c^{-0.3}\, p'\, 10^{-6} + 8\, e\, I_D\, 10^{-4}$

elastic limiting strain $\gamma_e = 0.012\,\gamma_{ref} + 2.10^{-6}$

Section 2: Plasticity theory

This section is common with the Soil Mechanics Databook supporting modules 3D1 and 3D2. Undrained shear strength ('cohesion' in a Tresca material) is denoted by s_u rather than c_u.

2.1 Plasticity: Tresca material, $\tau_{max} = s_u$

Limiting stresses

Tresca $\quad |\sigma_1 - \sigma_3| = q_u = 2s_u$

von Mises $\quad (\sigma_1 - p)^2 + (\sigma_2 - p)^2 + (\sigma_3 - p)^2 = \frac{2}{3} q_u^2 = 2s_u^2$

q_u= undrained triaxial compression strength; s_u= undrained plane shear strength.

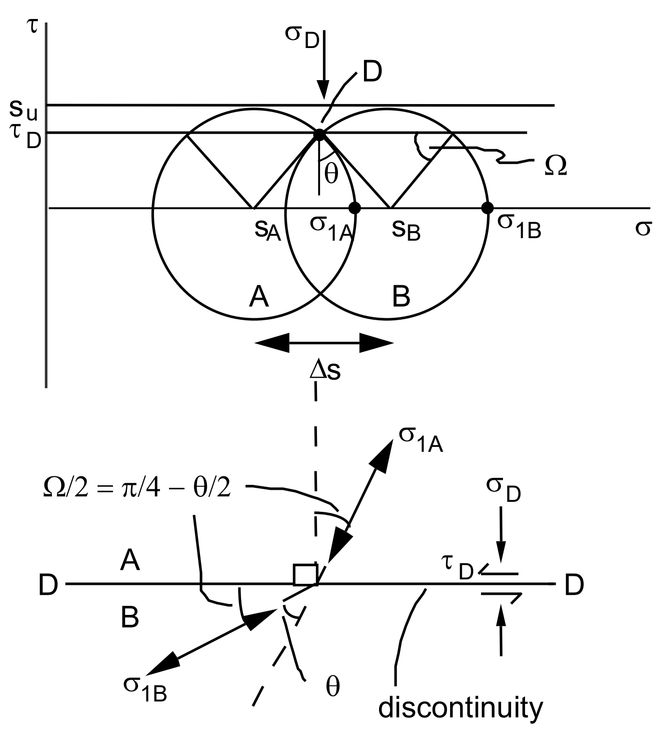
Dissipation per unit volume in plane strain deformation following either Tresca or von Mises,

$$\delta D = s_u \delta \varepsilon_\gamma$$

For a relative displacement x across a slip surface of area A mobilising shear strength s_u, this becomes

$$D = A s_u x$$

2.2 Stress conditions across a discontinuity:



Rotation of major principal stress

$$\theta = \pi/2 - \Omega$$

$$s_B - s_A = \Delta s = 2s_u \sin\theta$$
$$\sigma_{1B} - \sigma_{1A} = 2s_u \sin\theta$$

In limit with $\theta \rightarrow 0$

$$ds = 2s_u \, d\theta$$

Useful example:

$$\theta = 30^\circ$$

$$\sigma_{1B} - \sigma_{1A} = s_u$$

$$\tau_D / s_u = 0.87$$

σ_{1A} = major principal stress in zone A

σ_{1B} = major principal stress in zone B

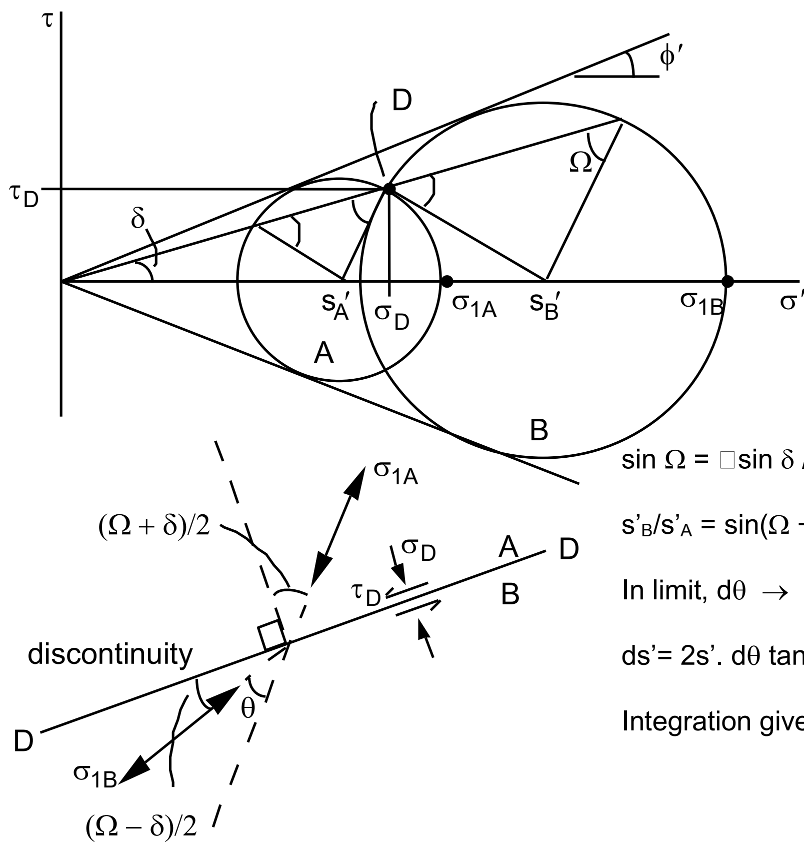
2.3 Plasticity: Coulomb material $(\tau/\sigma')_{max} = \tan\phi$

Limiting stresses

$$\sin\phi = (\sigma'_{1f} - \sigma'_{3f})/(\sigma'_{1f} + \sigma'_{3f}) = (\sigma_{1f} - \sigma_{3f})/(\sigma_{1f} + \sigma_{3f} - 2u)$$

where σ'_{1f} and σ'_{3f} are the major and minor principal effective stresses at failure, σ_{1f} and σ_{3f} are the major and minor principal total stresses at failure, and u is the pore pressure.

2.4 Stress conditions across a discontinuity



Rotation of major principal stress

$\theta = \pi/2 - \Omega$

σ_{1A} = major principal stress in zone A

σ_{1B} = major principal stress in zone B

$\tan\delta = \tau_D / \sigma'_D$

$\sin\Omega = \sin\delta / \sin\phi'$

$s'_B/s'_A = \sin(\Omega + \delta) / \sin(\Omega - \delta)$

In limit, $d\theta \to 0$ and $\delta \to \phi'$

$ds' = 2s'.\, d\theta \tan\phi'$

Integration gives $s'_B/s'_A = \exp(2\theta \tan\phi')$

Section 3: Bearing capacity of shallow foundations

3.1 Tresca soil, with undrained strength s_u

3.1.1 Vertical loading

The vertical bearing capacity, q_f, of a shallow foundation for undrained loading (Tresca soil) is:

$$\frac{V_{ult}}{A} = q_f = s_c d_c N_c s_u + \gamma h$$

V_{ult} and A are the ultimate vertical load and the foundation area, respectively. h is the embedment of the foundation base and γ (or γ') is the appropriate density of the overburden.

The exact bearing capacity factor N_c for a plane strain surface foundation (zero embedment) on uniform soil is:

$N_c = 2 + \pi$ (Prandtl, 1921)

Shape correction factor:
For a rectangular footing of length L and breadth B (Eurocode 7):

$s_c = 1 + 0.2\ B / L$

The exact solution for a rough circular foundation (B/L=1) is $q_f = 6.05 s_u$, hence $s_c = 1.18 \sim 0.2$.

Embedment correction factor:
A fit to Skempton's (1951) embedment correction factors, for an embedment of h, is:

$d_c = 1 + 0.33 \tan^{-1} (h/D)$ (or h/B for a strip or rectangular foundation)

3.1.2 Combined V-H loading

A curve fit to Green's lower bound plasticity solution for V-H loading is:

If $V/V_{ult} > 0.5$: $\frac{V}{V_{ult}} = \frac{1}{2} + \frac{1}{2}\sqrt{1 - \frac{H}{H_{ult}}}$ or $\frac{H}{H_{ult}} = 1 - \left(2\frac{V}{V_{ult}} - 1\right)^2$

If $V/V_{ult} < 0.5$: $H = H_{ult} = Bs_u$

3.1.3 Combined V-H-M loading

With lift-off: combined Green-Meyerhof ($V\rho_{ult}$ = bearing capacity of effective area B-e)

If $V/V\rho_{ult} < 0.5$: $\frac{H}{H_{ult}} = \left(1 - 2\frac{M}{VB}\right)$

Without lift-off: $\left(\frac{V}{V_{ult}}\right)^2 + \left[\frac{M}{M_{ult}}\left(1 - 0.3\frac{H}{H_{ult}}\right)\right]^2 + \left|\left(\frac{H}{H_{ult}}\right)^3\right| - 1 = 0$ (Taiebat & Carter 2000)

3.2 Frictional (Coulomb) soil, with friction angle ϕ

3.2.1 Vertical loading

The vertical bearing capacity, q_f, of a shallow foundation under drained loading (Coulomb soil) is:

$$\frac{V_{ult}}{A} = q_f = s_q N_q \sigma'_{v0} + s_\gamma N_\gamma \frac{\gamma' B}{2}$$

The bearing capacity factors N_q and N_γ account for the capacity arising from surcharge and self-weight of the foundation soil respectively. σ'_{v0} is the in situ effective stress acting at the level of the foundation base.

For a strip footing on weightless soil, the exact solution for N_q is:

$N_q = \tan^2(\pi/4 + \phi/2)\, e^{(\pi \tan \phi)}$ (Prandtl 1921)

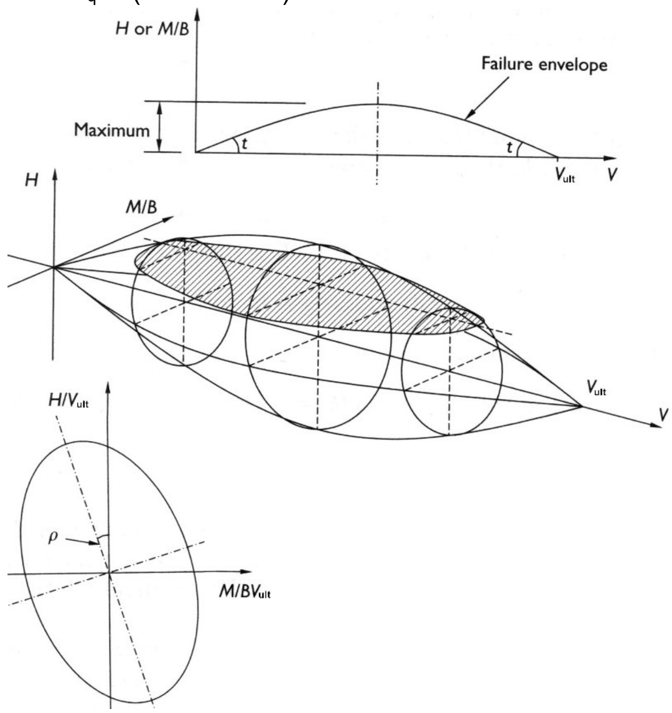
An empirical relationship to estimate N_γ from N_q is (Eurocode 7):

$N_\gamma = 2\,(N_q - 1) \tan \phi$

Curve fits to exact solutions for $N_\gamma = f(\phi)$ are (Davis & Booker 1971):

Rough base: $N_\gamma = 0.1054\, e^{9.6\phi}$

Smooth base: $N_\gamma = 0.0663\, e^{9.3\phi}$



Shape correction factors:

For a rectangular footing of length L and breadth B (Eurocode 7):

$s_q = 1 + (B \sin \phi) / L$
$s_\gamma = 1 - 0.3\, B / L$

For circular footings assume L = B.

3.2.2 Combined V-H loading

The Green/Sokolovski lower bound solution gives a V-H failure surface.

3.2.3 Combined V-H-M loading

(with lift-off- drained conditions- see failure surface shown above)

$$\left[\frac{H/V_{ult}}{t_h}\right]^2 + \left[\frac{M/BV_{ult}}{t_m}\right]^2 + \left[\frac{2C(M/BV_{ult})(H/V_{ult})}{t_h t_m}\right] = \left[\frac{V}{V_{ult}}\left(1 - \frac{V}{V_{ult}}\right)\right]^2$$

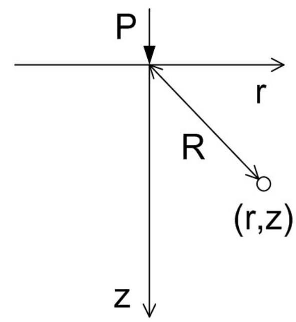
where $C = \tan\left(\frac{2\rho(t_h - t_m)(t_h + t_m)}{2 t_h t_m}\right)$ (Butterfield & Gottardi 1994)

Typically, t_h~0.5, t_m~0.4 and ρ~15°. t_h is the friction coefficient, $H/V = \tan \phi$, during sliding.

Section 4: Settlement of shallow foundations

4.1 Elastic stress distributions below point, strip and circular loads

Point loading (Boussinesq solution)



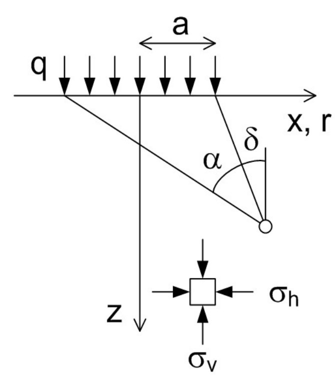
Vertical stress $$\sigma_z = \frac{3Pz^3}{2\pi R^5}$$

Radial stress $$\sigma_r = \frac{P}{2\pi R^2}\left[\frac{3r^2 z}{R^3} - \frac{(1-2\nu)R}{R+z}\right]$$

Tangential stress $$\sigma_\theta = \frac{P(1-2\nu)}{2\pi R^2}\left[\frac{R}{R+z} - \frac{z}{R}\right]$$

Shear stress $$\tau_{rz} = \frac{3Prz^2}{2\pi R^5}$$

Uniformly-loaded strip

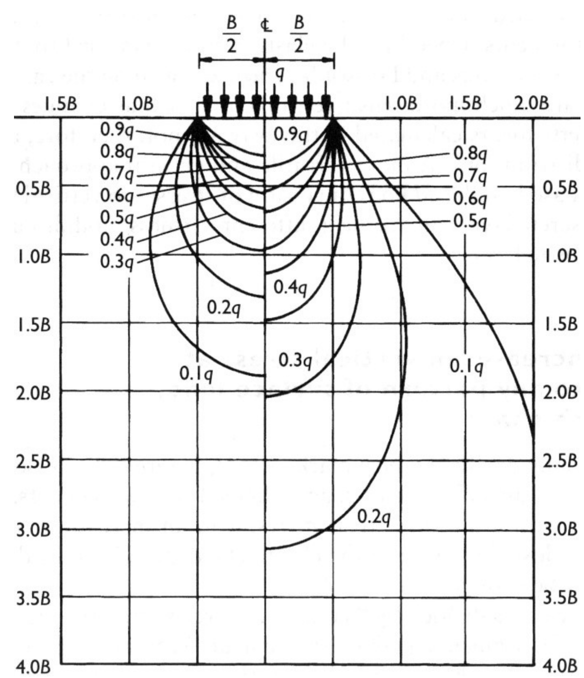


Vertical stress $$\sigma_v = \frac{q}{\pi}\left[\alpha + \sin\alpha\cos(\alpha+2\delta)\right]$$

Horizontal stress $$\sigma_h = \frac{q}{\pi}\left[\alpha - \sin\alpha\cos(\alpha+2\delta)\right]$$

Shear stress $$\tau_{vh} = \frac{q}{\pi}\sin\alpha\sin(\alpha+2\delta)$$

Principal stresses



$$\sigma_1 = \frac{q}{\pi}(\alpha + \sin\alpha) \quad \sigma_3 = \frac{q}{\pi}(\alpha - \sin\alpha)$$

Uniformly-loaded circle

(on centerline, r=0)

Vertical stress

$$\sigma_v = q\left[1-\left(\frac{1}{1+(a/z)^2}\right)^{\frac{3}{2}}\right]$$

Horizontal stress

$$\sigma_h = \frac{q}{2}\left[(1+2\nu)-\frac{2(1+\nu)z}{(a^2+z^2)^{1/2}}+\frac{z^3}{(a^2+z^2)^{3/2}}\right]$$

Contours of vertical stress below uniformly-loaded circular (left) and strip footings (right)

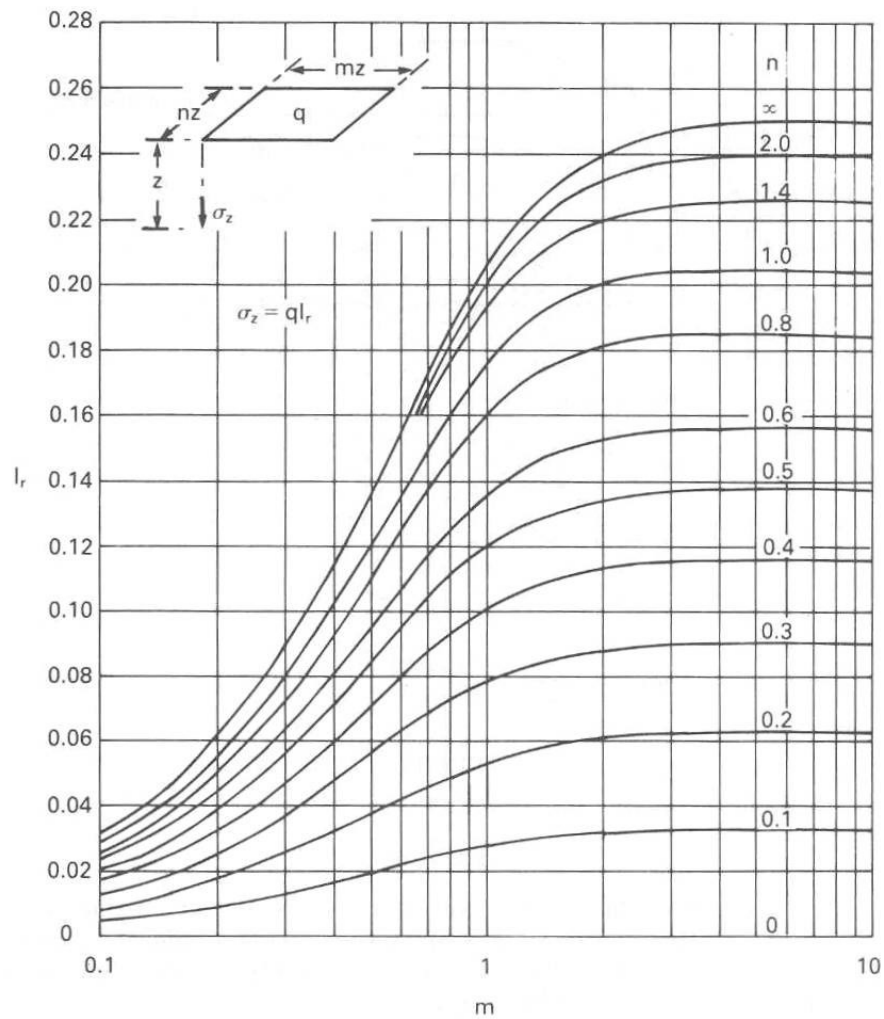
4.2 Elastic stress distribution below rectangular area

The vertical stress, σ_z, below the corner of a uniformly-loaded rectangle (L × B) is:

$$\sigma_z = I_r q$$

I_r is found from m (=L/z) and n (=B/z) using Fadum's chart or the expression below (L and B are interchangeable), which are from integration of Boussinesq's solution.

$$I_r = \frac{1}{4\pi}\left[\frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2+m^2n^2+1}\left(\frac{m^2+n^2+2}{m^2+n^2+1}\right)+\tan^{-1}\left(\frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2-m^2n^2+1}\right)\right]$$



Influence factor, I_r, for vertical stress under the corner of a uniformly-loaded rectangular area (Fadum's chart)

4.3 Elastic solutions for surface settlement

4.3.1 Isotropic, homogeneous, elastic half-space (semi-infinite)

Point load (Boussinesq solution)

Settlement, w, at distance s: $w(s) = \frac{1}{2\pi}\frac{(1-\nu)}{G}\frac{P}{s}$

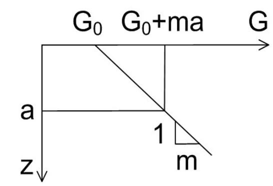
Circular area (radius a), uniform soil

Uniform load: central settlement: $w_o = \frac{(1-\nu)}{G} qa$

edge settlement: $w_e = \frac{2}{\pi}\frac{(1-\nu)}{G} qa$

Rigid punch: (q_{avg}= V/πa^2) $w_r = \frac{\pi}{4}\frac{(1-\nu)}{G} q_{avg} a$

Circular area, heterogeneous soil



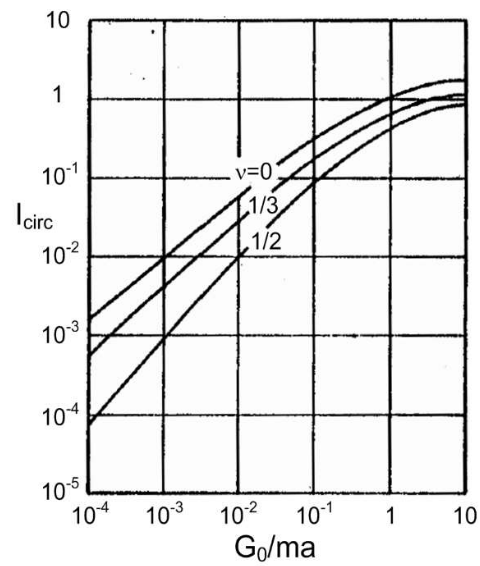
For G_0= 0, ν= 0.5:

w= q/2m under loaded area of any shape
w= 0 outside loaded area

For G_0> 0, central settlement:

$$w_o = \frac{qa}{2G_0} I_{circ}$$

For ν= 0.5, $w_o \approx \frac{qa}{2(G_0 + ma)}$



Rectangular area, uniform soil

Uniform load, corner settlement:

$$w_c = \frac{(1-\nu)}{G}\frac{qB}{2} I_{rect}$$

Where I_{rect} depends on the aspect ratio, L/B:

L/B	I_{rect}	L/B	I_{rect}	L/B	I_{rect}	L/B	I_{rect}
1	0.561	1.6	0.698	2.4	0.822	5	1.052
1.1	0.588	1.7	0.716	2.5	0.835	6	1.110
1.2	0.613	1.8	0.734	3	0.892	7	1.159
1.3	0.636	1.9	0.750	3.5	0.940	8	1.201
1.4	0.658	2	0.766	4	0.982	9	1.239
1.5	0.679	2.2	0.795	4.5	1.019	10	1.272

Rigid rectangle: $w_r = \frac{(1-\nu)}{G}\frac{q_{avg}\sqrt{BL}}{2} I_{rgd}$ where I_{rgd} varies from 0.9→0.7 for L/B = 1-10.

Note: $G = \frac{E}{2(1+\nu)}$ where ν= Poisson's ratio, E= Young's modulus.

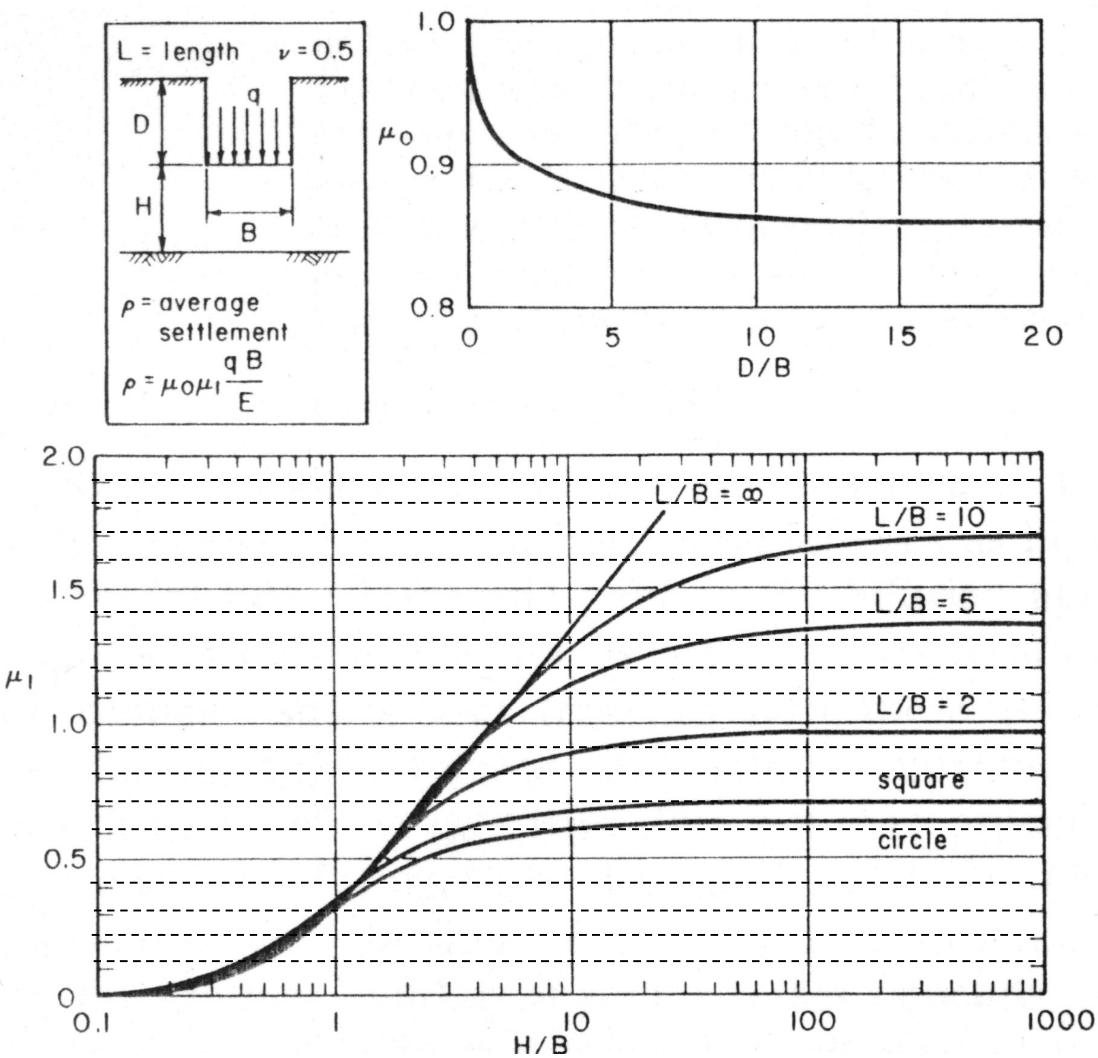
4.3.2 Isotropic, homogeneous, elastic finite space

Elastic layer of finite thickness

The mean settlement of a uniformly loaded foundation embedded in an elastic layer of finite thickness can be found using the charts below, for $\nu \sim 0.5$.

$$w_{avg} = \mu_0 \mu_1 \frac{qB}{E} \qquad\qquad E = 2G(1+\nu)$$

The influence factor μ_1 accounts for the finite layer thickness. The influence factor μ_0 accounts for the embedded depth.



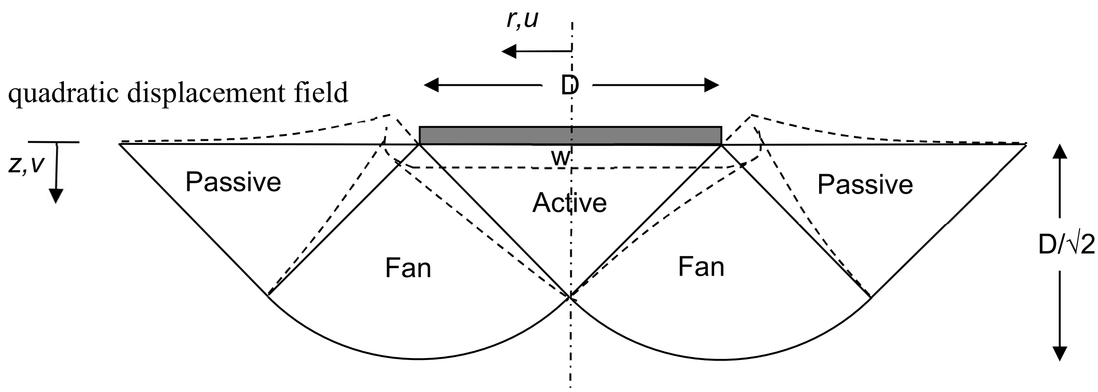
Average immediate settlement of a uniformly loaded finite thickness layer

Christian & Carrier (1978) Janbu, Bjerrum and Kjaernsli's chart reinterpreted. Canadian Geotechnical Journal (15) 123-128.

4.4 Mobilizable Strength Design (MSD) solutions

Rigid circular foundation on incompressible half-space

(Osman & Bolton, 2005)



Vertical bearing stress q

Average shear strain within deformation mechanism: $\gamma_{mob} = M_c\ w/D = 1.35\ w/D$

Average shear stress mobilized within mechanism: $\tau_{mob} = q / N_c = q / 5.9$

Representative depth to identify shear stress-strain behaviour: $z_{rep} = 0.3D$

If the representative soil test data fits: $\tau_{mob} = f(\gamma_{mob})$

Assume that the foundation load test data would fit: $(q/5.9) = f(1.35\ w/D)$

NB: this will underestimate w/D as $q \rightarrow 5.9\ s_u$, due to local strain concentrations

Horizontal or Moment loading

See Osman et al. (2007) Geotechnique 57 (9) 729-737

Section 5: Bearing capacity of deep foundations

5.1 Axial capacity: API (2000) design method for driven piles

5.1.1 Sand

Unit shaft resistance: $\tau_{sf} = \sigma'_{hf} \tan\delta = K\sigma'_{v0} \tan\delta \leq \tau_{s,lim}$

Closed-ended piles: $K = 1$
Open-ended piles: $K = 0.8$

Unit base resistance: $q_b = N_q \, \sigma'_{vo} < q_{b,limit}$

Soil category	Soil density	Soil type	Soil-pile friction angle, δ (°)	Limiting value $\tau_{s,lim}$ (kPa)	Bearing capacity factor, N_q	***Limiting value, $q_{b,lim}$ (MPa)***
1	Very loose Loose Medium	Sand Sand-silt Silt	15	50	8	1.9
2	Loose Medium Dense	Sand Sand-silt Silt	20	75	12	2.9
3	Medium Dense	Sand Sand-silt	25	85	20	4.8
4	Dense Very dense	Sand Sand-silt	30	100	40	9.6
5	Dense Very dense	Gravel Sand	35	115	50	12

API (2000) recommendations for driven pile capacity in sand

5.1.2 Clay

American Petroleum Institute (API) (2000) guidelines for driven piles in clay.

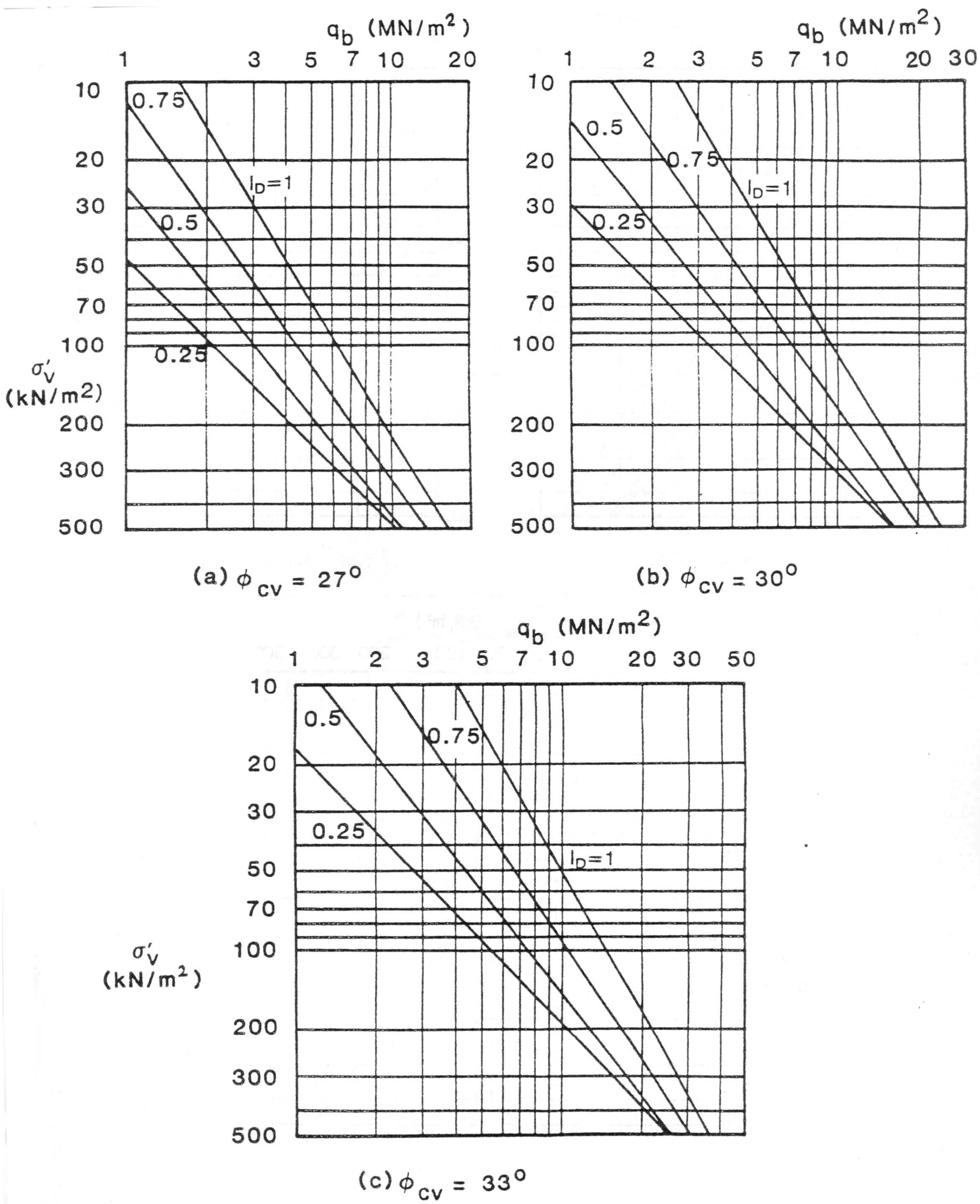
Unit shaft resistance: $$\alpha = \frac{\tau_s}{s_u} = 0.5 \cdot \mathrm{Max}\left[\left(\frac{\sigma'_{vo}}{s_u}\right)^{0.5}, \left(\frac{\sigma'_{vo}}{s_u}\right)^{0.25}\right]$$

It is assumed that equal shaft resistance acts inside and outside open-ended piles.

Unit base resistance: $q_b = N_c \, s_u$ $\quad N_c = 9.$

5.2 Axial capacity: base resistance in sand using Bolton's stress dilatancy

Unit base resistance, q_b, is expressed as a function of relative density, I_D, constant volume (critical state) friction angle, ϕ_{cv}, and in situ vertical effective stress, σ'_v.



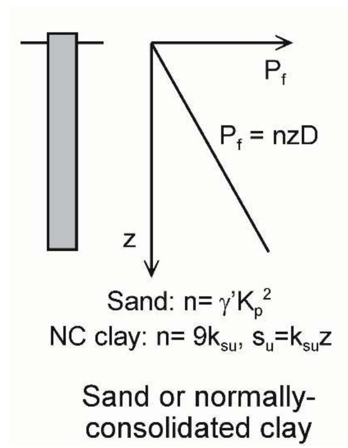
Design charts for base resistance in sand
(Randolph 1985, Fleming et al 1992)

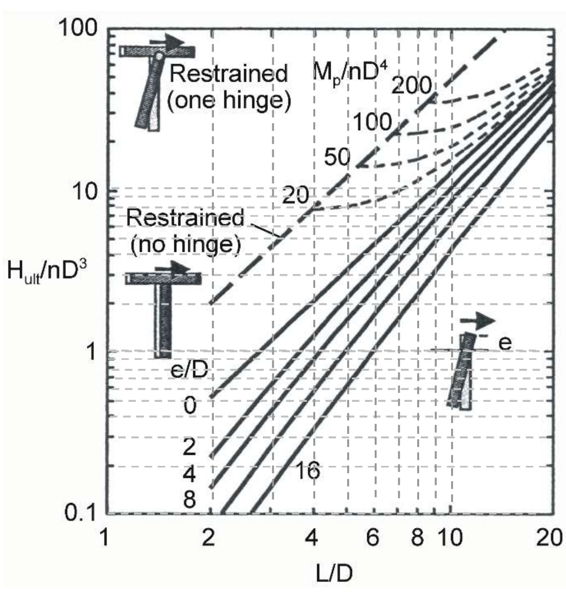
5.3 Lateral capacity: linearly increasing lateral resistance with depth

Lateral soil resistance (force per unit length), $P_u = nzD$

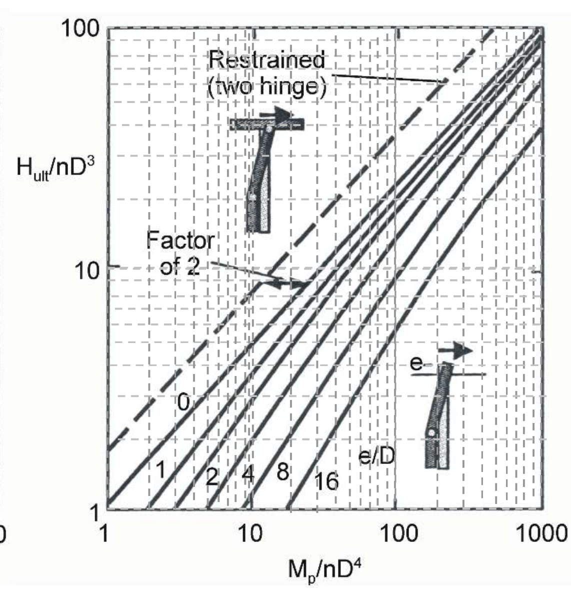
In sand, $n = \gamma' K_p^2$
In normally consolidated clay with strength gradient k; $s_u = kz$; $n=9k$

H_{ult}	ultimate horizontal load on pile
M_p	plastic moment capacity of pile
D	pile diameter
L	pile length
e	load level above pile head (=M/H for H-M pile head loading)
γ'	effective unit weight
K_p	passive earth pressure coefficient, $K_p = (1+\sin\phi)/(1-\sin\phi)$





Short pile failure mechanism



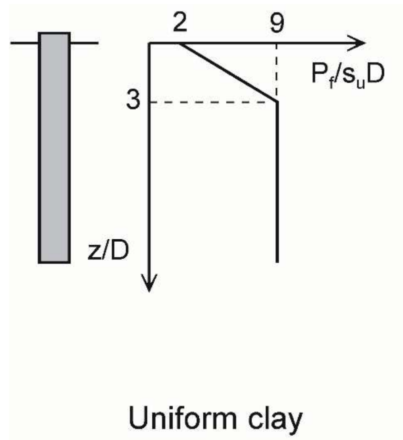
Long pile failure mechanism

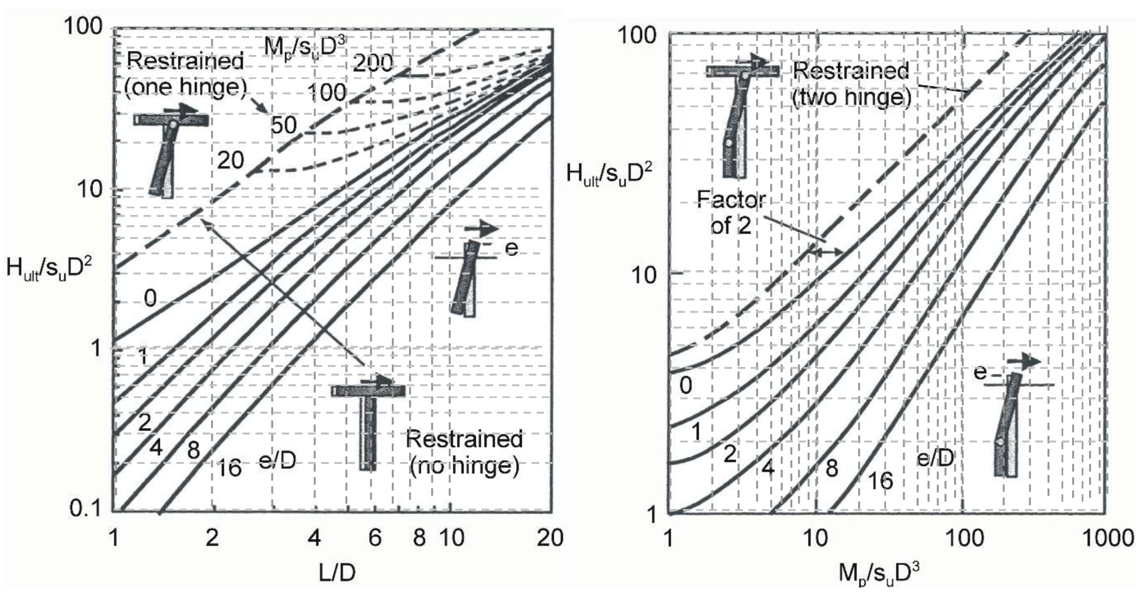
Lateral pile capacity
(linearly increasing lateral resistance with depth)

5.4 Lateral capacity: uniform clay

Lateral soil resistance (force per unit length), P_u, increases from $2s_uD$ at surface to $9s_uD$ at 3D depth then remains constant.

H_{ult} ultimate horizontal load on pile
M_p plastic moment capacity of pile
D pile diameter
L pile length
e load level above pile head
(=M/H for H-M pile head loading)
s_u undrained shear strength





Short pile failure mechanism

Long pile failure mechanism

Lateral pile capacity
(uniform clay lateral resistance profile)

Section 6: Settlement of deep foundations

6.1 Settlement of a rigid pile

Shaft response:

Equilibrium:

$$\tau = \tau_s \frac{R}{r}$$

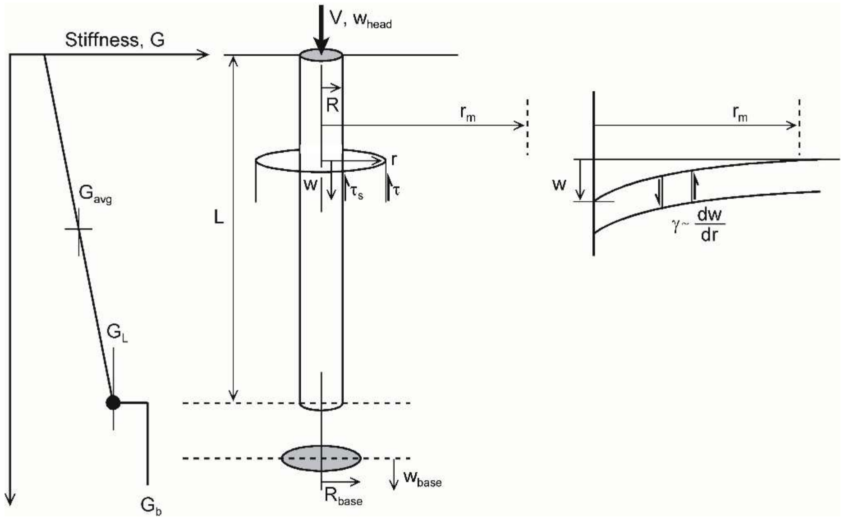
Compatibility:

$$\gamma \approx \frac{dw}{dr}$$

Elasticity:

$$\frac{\tau}{\gamma} = G$$

Integrate to magical radius, r_m, for shaft stiffness, τ_s/w.



Nomenclature for settlement analysis of single piles

Combined response of base (rigid punch) and shaft:

$$\frac{V}{w_{head}} = \frac{Q_b}{w_{base}} + \frac{Q_s}{w} \qquad \frac{V}{w_{head}} = \frac{4R_{base}G_{base}}{1-\nu} + \frac{2\pi L G_{avg}}{\zeta}$$

$$\frac{V}{w_{head}DG_L} = \frac{2}{1-\nu}\frac{G_{base}}{G_L}\frac{D_{base}}{D} + \frac{2\pi}{\zeta}\frac{G_{avg}}{G_L}\frac{L}{D} \qquad \frac{V}{w_{head}DG_L} = \frac{2}{1-\nu}\frac{\eta}{\xi} + \frac{2\pi}{\zeta}\rho\frac{L}{D}$$

These expressions are simplified using dimensionless variables:

Base enlargement ratio, eta $\eta = R_{base}/R = D_{base}/D$ Slenderness ratio L/D

Stiffness gradient ratio, rho $\rho = G_{avg}/G_L$ Base stiffness ratio, xi $\xi = G_L/G_{base}$

It is often assumed that the dimensionless zone of influence, $\zeta = \ln(r_m/R) = 4$.

More precise relationships, checked against numerical analysis are:

$$\zeta = \ln\left\{\{0.5 + (5\rho(1-\nu) - 0.5)\xi\}\frac{L}{D}\right\} \qquad \text{for } \xi=1: \qquad \zeta = \ln\left\{5\rho(1-\nu)\frac{L}{D}\right\}$$

6.2 Settlement of a compressible pile

$$\frac{V}{w_{head}DG_L} = \frac{\dfrac{2\eta}{(1-\nu)\xi} + \rho\dfrac{2\pi}{\zeta}\dfrac{\tanh\mu L}{\mu L}\dfrac{L}{D}}{1 + \dfrac{1}{\pi\lambda}\dfrac{8\eta}{(1-\nu)\xi}\dfrac{\tanh\mu L}{\mu L}\dfrac{L}{D}} \qquad \text{where } \mu = \frac{\sqrt{8/\zeta\lambda}}{D} \quad \text{Pile compressibility}$$

$\lambda = E_p/G_L$ Pile-soil stiffness ratio

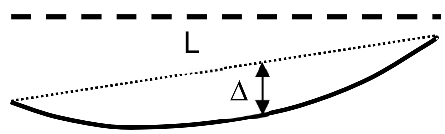
Pile head stiffness, $\frac{V}{w_{head}}$, is maximum when $L \geq 1.5D\sqrt{\lambda}$

Section 7: Damage to buildings from differential settlement

Relative displacement Δ/L:

L is the length of building segment with consistent sagging or hogging

Δ is the maximum settlement of the deformed segment relative to chord L



Distortion and maximum tensile strain ε_{max} in elastic beams of various E/G and L/H:

$\frac{\Delta}{L\,\varepsilon_{max}} \approx$ 1.0 to 1.5 diagonally in end panels due to shear

0.75 to 1.0 longitudinally due to sagging beam

0.25 to 0.5 longitudinally due to hogging beam

Onset of visible (~ 0.1mm) cracks in brick or blockwork walls: $\varepsilon_{max} \approx 0.75\ .10^{-3}$

(Burland & Wroth, 1974)

Categories of associated building damage:

Cat.	Limit	Relative displacement	Description	Action
0	-	$\Delta/L \leq 0.5\ 10^{-3}$	negligible	none
1	SLS	$0.5\ 10^{-3} < \Delta/L \leq 0.75\ 10^{-3}$	very slight	redecorate interior
2	SLS	$0.75\ 10^{-3} < \Delta/L \leq 1.5\ 10^{-3}$	slight	+ some repointing
3	SLS	$1.5\ 10^{-3} < \Delta/L \leq 3.\ 10^{-3}$	moderate	+ significant repointing etc
4	ULS	$3.\ 10^{-3} < \Delta/L \leq 10^{-2}$	severe	shore; consider demolition
5	ULS	$10^{-2} < \Delta/L$	very severe	demolish

(Boscardin & Cording, 1989)

Section 8: Cylindrical cavity expansion

Expansion $\delta A = A - A_o$ caused by increase of pressure $\delta\sigma_c = \sigma_c - \sigma_o$

At radius r: small displacement $\rho = \dfrac{\delta A}{2\pi r}$

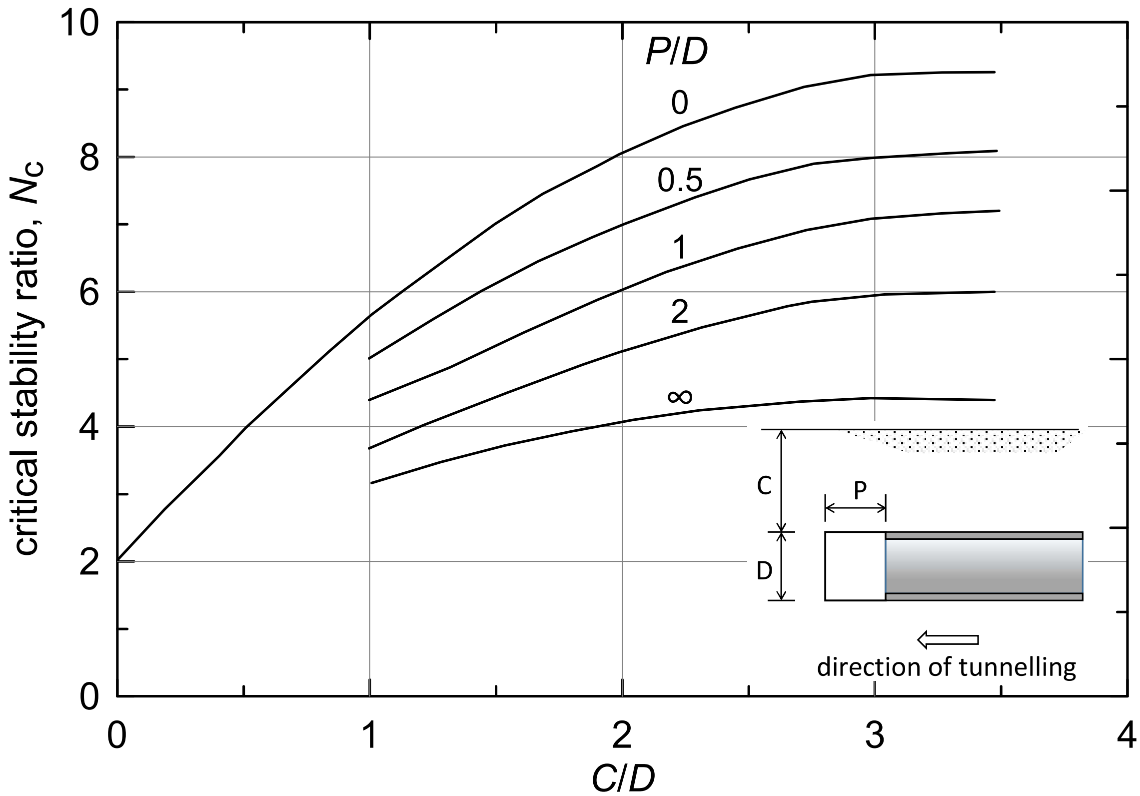
small shear strain $\gamma = \dfrac{2\rho}{r}$

Radial equilibrium: $r\dfrac{d\sigma r}{dr} + \sigma_r - \sigma_\theta = 0$

Elastic expansion (small strains) $\delta\sigma_c = G\dfrac{\delta A}{A}$

zUndrained plastic-elastic expansion $\delta\sigma_c = c_u\left[1 + \ln\dfrac{G}{c_u} + \ln\dfrac{\delta A}{A}\right]$

Section 9: Tunnel Face Stability



Section 10: Ground Movements around tunnels

$$w = w_{max} e^{-\frac{1}{2}\left(\frac{x}{i}\right)^2}$$

$$i = K z_0$$

Where z_0 is the depth of the axis of a tunnel.

K is 0.65 for soft clay, 0.45 for stiff clay and 0.25 for sand above the water table.