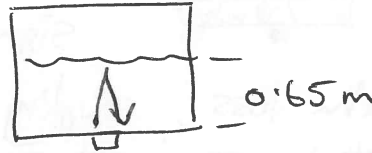


4B13 CRIB 2018

1(a)



$$\text{Transit time for pulse-echo} = \frac{2 \times 0.65}{1324} + \frac{2 \times 0.003}{5960}$$

$$= 0.983 \text{ ms} \quad [10\%]$$

(b)

$$\begin{aligned} Z_{\text{air}} &= 340 \text{ Rayls} \\ Z_{\text{oil}} &= \rho v = 1.072 \text{ M Rayls} \\ Z_{\text{steel}} &= 45.3 \text{ M Rayls} \\ Z_{\text{PZT}} &= 30.0 \text{ M Rayls} \\ Z_{\text{water}} &= 1.5 \text{ M Rayls} \end{aligned}$$

$$P_{\text{PZT}} = \sqrt{Z/R} = 144/500 = 0.288 \text{ W}$$

$$\text{coupling PZT} \leftrightarrow \text{steel} = \frac{4Z_p Z_s}{(Z_p + Z_s)^2} = 0.959$$

$$\text{coupling steel} \leftrightarrow \text{oil} = 0.090$$

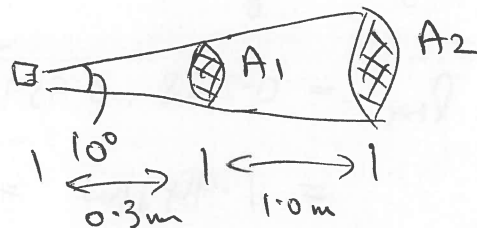
$$\text{reflection oil} \leftrightarrow \text{air} = \frac{(Z_o - Z_a)^2}{(Z_o + Z_a)^2} = 0.999$$

$$\text{Attenuation over } 1.3 \text{ m in oil} = 0.91 \text{ dB} = \times 0.811$$

$$0.3 \text{ m in oil} = 0.21 \text{ dB} = \times 0.953$$

$$6 \text{ mm in steel} = 0.138 \text{ dB} = \times 0.969$$

Beam angle spread,



$$A_1 = 21.6 \text{ cm}^2$$

$$A_2 = 406 \text{ cm}^2$$

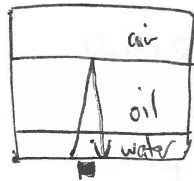
$$\text{For } 0.65 \text{ m, } P_{\text{rec}} = \frac{V_r^2}{R} = 0.288 \cdot 0.959 \cdot 0.09^2 \cdot 0.999 \cdot 0.25^2 \cdot 0.811 \cdot 0.969 \cdot \frac{\pi}{406} = 0.815 \text{ } \mu\text{W}$$

$$\therefore V_r = 20.2 \text{ mV loaded} = 40.4 \text{ mV open ct.}$$

$$\text{For } 0.15 \text{ m, } P_{\text{rec}} = 0.288 \cdot 0.959 \cdot 0.09^2 \cdot 0.999 \cdot 0.25^2 \cdot 0.953 \cdot 0.969 \cdot \frac{\pi}{21.6} = 18.0 \text{ } \mu\text{W}$$

$$[50\%] \Rightarrow 0.09 \text{ V loaded} \Rightarrow 0.19 \text{ V open ct}$$

1 (c)



Coupling steel $\leftrightarrow$ water = 0.124
 VS. 0.09 steel $\leftrightarrow$ oil so
 Signal is larger by $\times 1.38$

Extra reflection loss, coupling oil $\leftrightarrow$ water = 0.972
 so net increase in ultrasonic power each way = $\times 1.34$

Transit time now: $\frac{2 \times 0.003}{5960} + \frac{2 \times 0.6}{1324} + \frac{2 \times 0.05}{1500} = 0.974 \text{ ms}$

So, the ^{net} signal voltage will be larger by $\times 1.34$ (34%)
 and the timing 99% (so the level indicated will be
 wrong - it actually partially compensates for oil being
 sat on a layer of water). Reads 64.4 cm apparent
 oil, but is of course only 60 cm of oil. [20%]

(d) There will be a partial reflection from the
 oil-water interface of $\times 0.028$. water alt = 0.078
 $\phi 37.5 \text{ mm} \times 0.984$

Beam area @ water/oil



Magnitude of oil-water reflection signal:

$$P_{ref} = 0.288 \cdot 0.959^2 \cdot 0.124^2 \cdot 0.028 \cdot 0.984 \cdot \frac{\pi}{11} \cdot 0.25^2$$

$$= 1.94 \mu\text{W} = \frac{V_r^2}{R} \quad \therefore V_r \text{ open ct} = 62.3 \text{ mV}_{\text{water}}$$

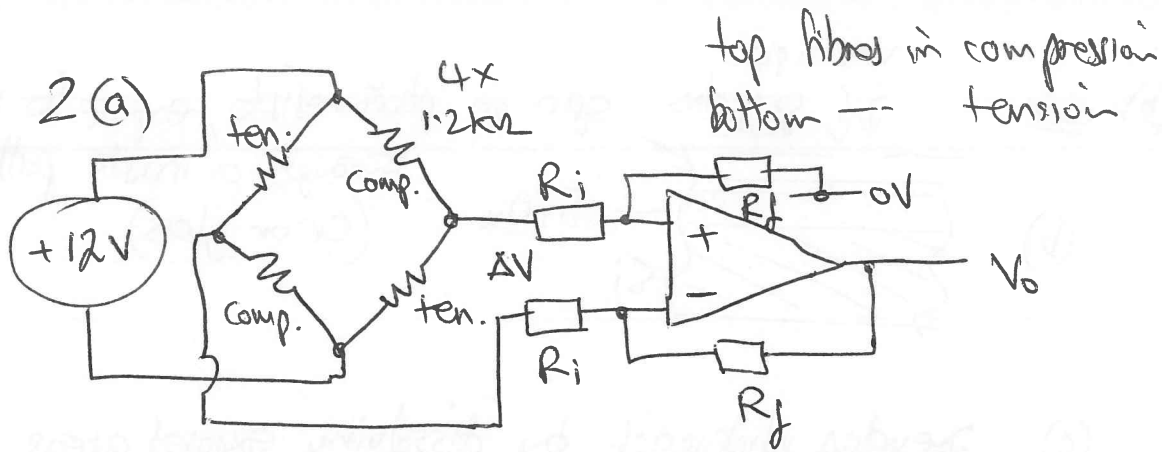
VS. $1.34 \times 40.4 \text{ mV} = 54.1 \text{ mV}$

i.e. rather similar signal levels, but

pulse-echo time for water = 0.068 ms.

oil.
(87%
of water
signal)

[20%]



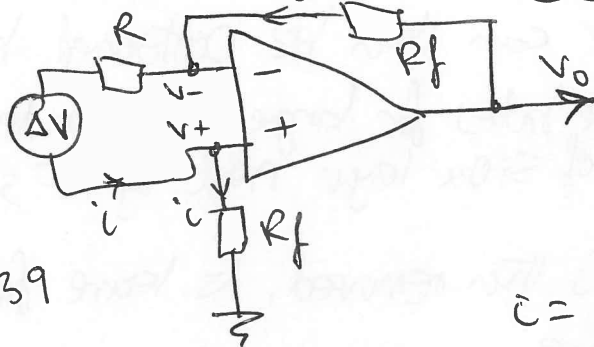
Connect fibre strain gauges in a bridge and input to a differential amplifier.

the ΔV signal for a full bridge = $\frac{\Delta R}{R} \cdot V_s$

$\therefore$ at 0.2% strain: $\Delta V = \frac{2}{1000} \cdot 1.5 \cdot 12 = 36 \text{ mV}$

with a source impedance of 1.2k Ω for simplicity we can make $R_i = 0$ then

$V_o = \frac{2 R_f}{1.2k} \Delta V$
(5) (0.036)



since: $V_- = V_+ = V$

$\frac{V_o - V}{R_f} = \frac{V}{R_f} \therefore V_o = 2V$

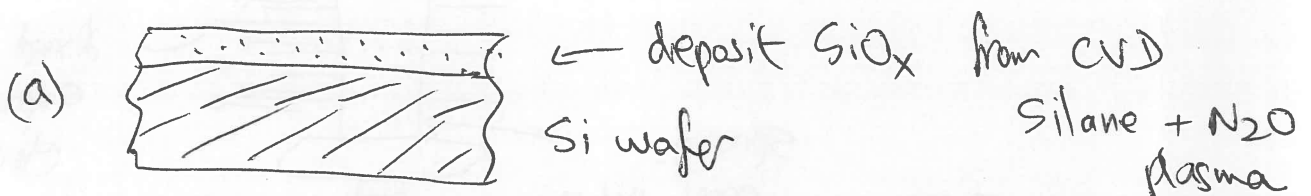
$V = \frac{\Delta V}{R} = \frac{V}{R_f} \therefore V_o = 2 \frac{R_f}{R} \Delta V$

$G = \frac{5}{0.036} = 139$

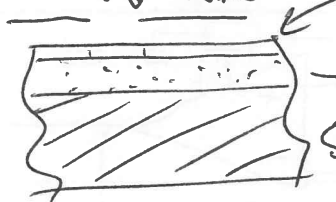
So, choose $R_f = 83.3 \text{ k}\Omega$

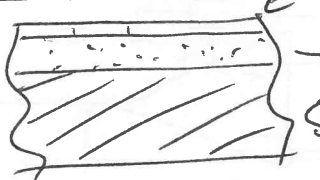
[25%]

- (b)(i) Surface micro-machining
- deposition of Si, SiO_2 , Si_3N_4
 - photolithography
 - wet chemical etching

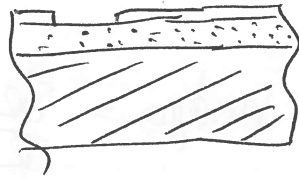


2(b) cont.

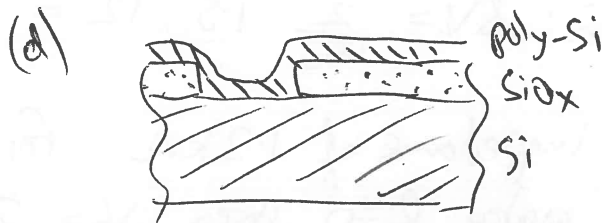
UV light
 spin on photoresist, expose to UV through a mask pattern (cronglass)

(b)  SiOx
Si

(c) develop photoresist by dissolving exposed areas in caustic solution eg: NaOH



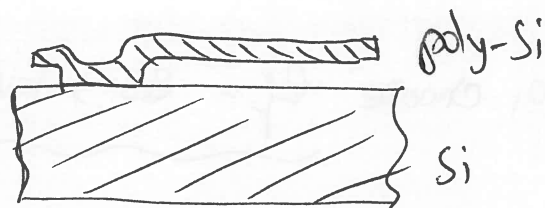
then etch in HF solution
 to dissolve SiOx
 or plasma etch in CF_4



then deposit poly-Si from SiH_4 CVD heat/plasma deposition

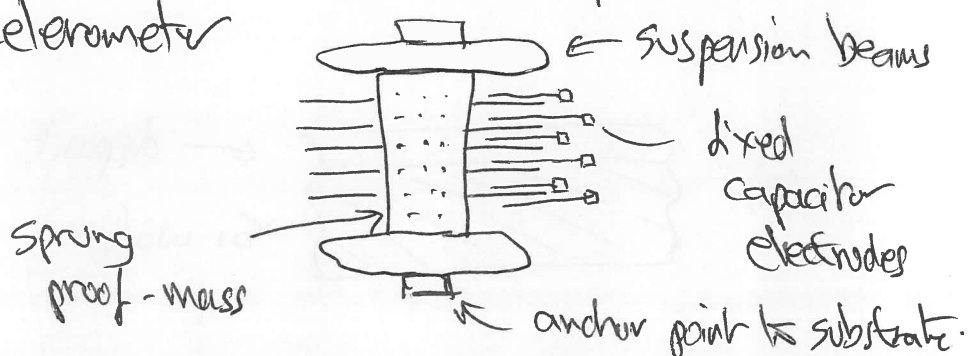
(e) Poly-Si layer can then be patterned by dry plasma etching (leave holes for large area under-etching) of SiOx layer next by HF solution

(f) SiOx layer is then removed, to leave free standing poly-Si structure



(g) can include SiN insulating layers and metal layers (deposit by sputtering/evaporation).

(h) to form accelerometer



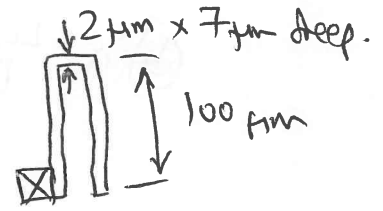
[20%]

2(b)(ii) Capacitor area = $100 \times 100 \times 10^{-6} \times 7 \times 10^{-6} = 1.4 \times 10^{-7} \text{ m}^2$
 gap = $1.5 \times 10^{-6} \text{ m}$

$\therefore$ Total capacitance = $\frac{A\epsilon_0}{d} = \underline{0.413 \text{ pF}}$ [10%]

(iii) Each hairpin tether spring:

Beam ends do not change angle



$\uparrow \xrightarrow{W} + \uparrow \xrightarrow{M} = \uparrow \xrightarrow{\delta}$
 for $d=0$

$M = \frac{WL}{2} \therefore \delta = \frac{WL^3}{3EI} - \frac{WL^3}{4EI} = \frac{WL^3}{12EI}$ Spring constant $k = W/\delta$

For a hairpin tether, there are 2 such beams, and 1 tether at each corner.

$E = 110 \text{ GN m}^{-2}, I = \frac{dw^3}{12}$

$\therefore k = \frac{12EI}{L^3} \times 4/2$
 $\nwarrow$
 $100 \times 10^{-6} \text{ m}$

$d = 7 \times 10^{-6} \text{ m}$
 $w = 2 \times 10^{-6} \text{ m}$

$\therefore$ Spring constant, k for proof mass = $\underline{12.3 \text{ N m}^{-1}}$ [20%]

(iv) Proof mass $\approx 1.96 \times 10^{-9} \text{ kg}$

$\therefore$ Deflection @ $100 \text{ ms}^{-2} = \frac{100 \times 1.96 \times 10^{-9}}{12.3} = 1.59 \times 10^{-8} \text{ m}$

$\therefore$ % change in air gap = $\frac{1.59 \times 10^{-8}}{1.5 \times 10^{-6}} \times 100 = 1.06\%$
 and capacitance [10%]

(v) $C = \frac{A\epsilon_0}{x}, E = \frac{1}{2} CV^2 = \frac{1}{2} \frac{A\epsilon_0}{x} V^2, F = \frac{dE}{dx} \text{ (force)}$

$\therefore F = -\frac{1}{2} \frac{A\epsilon_0 V^2}{x^2} = -\frac{1}{2} \frac{CV^2}{x} \text{ or } V = \sqrt{\frac{2Fx}{C}}$

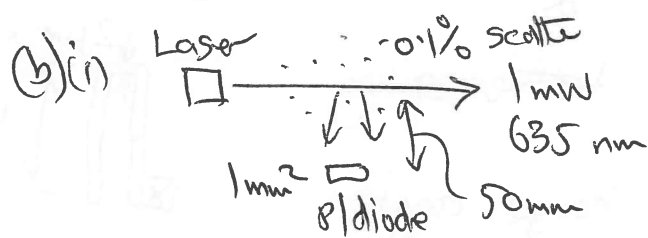
$\therefore V = \left(\frac{2 \times 100 \times 1.96 \times 10^{-9} \times 1.5 \times 10^{-6}}{0.413 \times 0.60 \times 10^{-12}} \right)^{1/2} = \underline{1.54 \text{ V}}$ [15%]

Closed-loop response gives better linearity, dynamic range and transient response - electrical feedback increases k .

$$3(a) \quad R = A e^{\beta/T}$$

$$1000 = A e^{3200/(20+273)} \quad \therefore A = 0.0181$$

$$\therefore @ 60^\circ\text{C} \quad R = 0.0181 e^{3200/(60+273)} = \underline{270 \Omega}$$



optical scattered [10%]

$$\therefore \text{power on p/diode} = 10^{-3} \times 10^{-3} \times \frac{1}{4\pi \cdot 50^2} = 31.8 \text{ pW}$$

Photodiode quantum effie = 60%

$$\text{photon energy @ 635 nm} \Rightarrow \frac{hc}{\lambda} = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{635 \times 10^{-9}} = 3.13 \times 10^{-19}$$

$$\therefore \text{photon flux detected} = \frac{31.8 \times 10^{-12}}{3.13 \times 10^{-19}} = 101.6 \times 10^6 \text{ photon/sec}$$

$$\therefore \text{photocurrent} = 1.6 \times 10^{-19} \times 101.6 \times 10^6 \times 0.60 = \underline{9.75 \text{ pA}} \quad [20\%]$$

(ii) $10 \text{ M}\Omega \parallel 10 \text{ nF}$ $f_{-3dB} = \frac{1}{2\pi RC} = 1.59 \text{ Hz}$

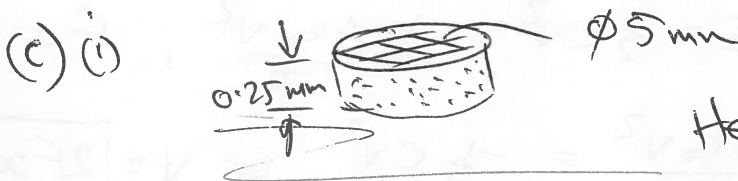
$$\text{Signal amplitude} = 10 \times 10^6 \times 9.75 \times 10^{-12} = \underline{97.5 \mu\text{V}}$$

$$\text{op amp noise} = 10 \times 10^6 \times 1.2 \times 10^{-12} \times \sqrt{1.59} = 15.1 \mu\text{V}_{\text{rms}}$$

$$\text{resistor thermal noise} = \sqrt{4kTRB} = 0.51 \mu\text{V}_{\text{rms}}$$

$$V_{n_{\text{total}}} = \sqrt{\sum v_n^2} = \sqrt{15.1^2 + 0.51^2} = 15.1 \mu\text{V}_{\text{rms}}$$

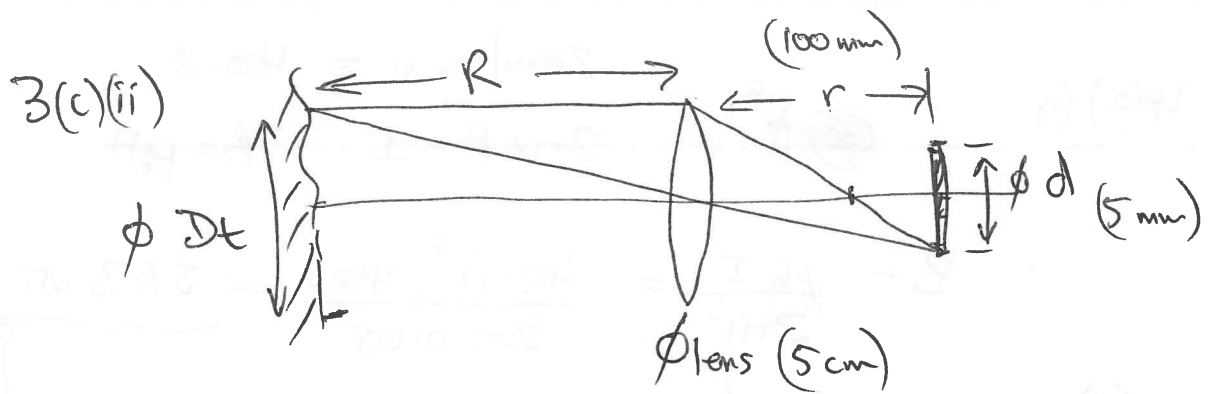
$$\therefore \text{S/N ratio} \approx \underline{6.5 : 1} \quad [25\%]$$



$$\text{Heat flow} = \frac{kA \Delta T}{d} = P_h$$

$$\text{Thermal rating} = \frac{\Delta T}{P_h} = \frac{d}{kA} = \frac{0.25 \times 10^{-3} \times 4}{0.05 \times \pi (5 \times 10^{-3})^2}$$

$$= \underline{255^\circ\text{C/W}} \quad [10\%]$$



Lambert's Law, $\delta W = \frac{W \cos \theta A \delta \omega}{\pi} = \frac{W}{\pi} \cdot \frac{\pi D_t^2}{4} \cdot \frac{\pi \phi_{\text{lens}}^2}{4 R^2}$

Now $\frac{D_t}{R} = \frac{d}{r} \therefore \delta W = \frac{W}{\pi} \cdot \frac{\pi^2}{16} \cdot \frac{d^2}{r^2} \cdot \phi_{\text{lens}}^2$

$\therefore \delta W = \epsilon \overset{5.67 \times 10^{-8}}{\sigma_{\text{sb}}} T^4 \overset{0.95}{\uparrow} \frac{\pi}{16} \frac{d^2 \phi_{\text{lens}}^2}{r^2} = 8.15 \times 10^{-4} W$
 $(60 + 273.2)$

$\therefore$ temperature rise of detector $= 8.15 \times 10^{-4} \times 255 = 0.208^\circ \text{C}$

and with $P_s = \frac{2.6 \times 1.38 \times 10^{-23}}{1.6 \times 10^{-19}} \ln \left(\frac{2 \times 10^{-3}}{5.6 \times 10^{-6}} \right) = 1.32 \text{ mV/K}$

$\therefore \text{Signal} = 0.208 \times 1.32 = 0.274 \text{ mV} \quad [20\%]$

(iii) Mass of detector $= \rho A d = 150 \cdot \frac{\pi (5 \times 10^{-3})^2}{4} \cdot 0.25 \times 10^{-3}$
 $= 7.36 \times 10^{-7} \text{ kg}$

$\frac{kAT}{d} = m c_p \dot{T}$ for heat flux to detector, a 1st order

D.F. with exponential soln. $\tau = \frac{m c_p d}{kA} = 0.225 \text{ secs.}$

So bandwidth $f \approx \frac{1}{2\pi\tau} = 0.71 \text{ Hz}$

$[15\%]$

4(a) (i)

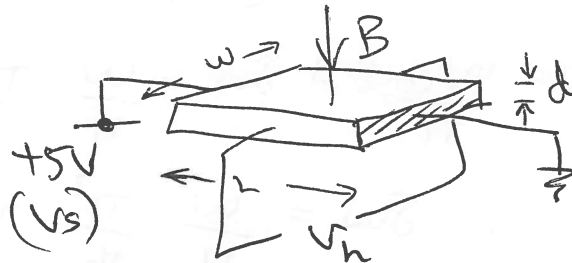


$$2\text{mm}/500\text{V} = 4000\text{A}$$

$$2\pi r H = I \quad B = \mu_0 H$$

$$\therefore B = \frac{\mu_0 I}{2\pi r} = \frac{4\pi \times 10^{-7} \cdot 4000}{2\pi \cdot 0.015} = 53.3\text{ mT} \quad [15\%]$$

(ii)



Force balance on electrons: $Bqv = qE$

$$\therefore B \frac{V_h}{L} \mu = \frac{v_n}{w}$$

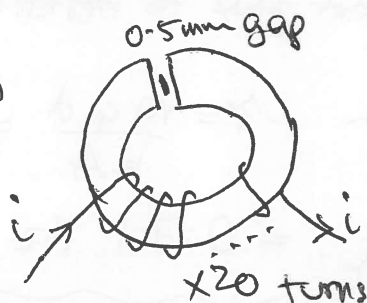
$$\therefore V_h = B \underbrace{\mu}_{0.16} \underbrace{\frac{w}{L}}_5 V_s = 0.8 B$$

$$\therefore \text{responsivity} = 0.8\text{ V/T}$$

$$\therefore \text{Signal} = 42.6\text{ mV} @ 4000\text{A} \quad [20\%]$$

(iii) To improve immunity to external influences, use another Hall sensor diametrically opposite and connect the signals differentially. As the cable produces a circular field the signals add but uniform external fields will cancel.  [10%]

(b) (i)



$$\sum Hl = Ni \quad \text{and} \quad B = \mu_0 H_a = \mu_0 \mu_r H_m$$

$$H_a g + H_m l = Ni = H_a g + H_a \frac{l}{\mu_r}$$

$$\therefore B = \frac{Ni \mu_0}{(g + \frac{l}{\mu_r})}, \quad \phi = BA$$

$$L = \frac{N\phi}{i} = N^2 \mu_0 A / (g + \frac{l}{\mu_r}) = 28.3\text{ }\mu\text{H} \quad [15\%]$$

4(b)(i) with $0.5 \text{ A} = 10 \text{ Atoms}$

$$B = \frac{N_i \mu_0}{(g + 4 \mu_r)} = \frac{10 \cdot 4\pi \times 10^{-7}}{0.533 \times 10^3} = 0.0236 \text{ T}$$

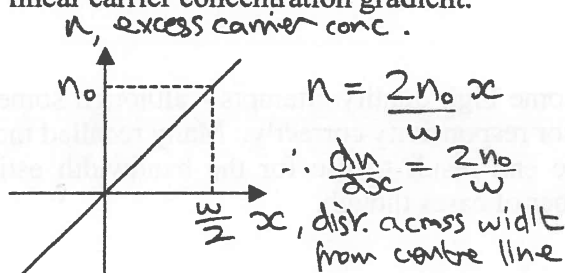
$$\Rightarrow V_{\text{signal}} = 0.0236 \times 0.80 \Rightarrow \underline{18.9 \text{ mV}} \quad [15\%]$$

(iii) Resistance of Hall sensor = $\frac{\rho L}{A} = \frac{10^{-3} \cdot L}{15 \times 10^{-6} \cdot L} = \underline{66.7 \Omega}$

$$\text{Thermal noise} = \sqrt{4kT R B}$$

So we need the bandwidth, B .

The response bandwidth may be estimated from considering the charge carrier transit time constant across the device, by diffusion. For simplicity, we shall assume a linear carrier concentration gradient.



carrier flux Fick's Law:-

$$f = -D \frac{dn}{dx}, \quad D = \frac{\mu kT}{q} = 4.14 \times 10^{-3}$$

N = total excess carriers / side:

$$N = Ld \int_0^{w/2} \frac{2n_0}{w} x dx = \frac{n_0 w L d}{4}$$

consider 1 side: $\frac{dN}{dt} = f L d = -D \frac{2n_0}{w} L d = -D \frac{2Ld}{w} \frac{4N}{w L d}$

$\therefore \frac{dN}{dt} = -\frac{8DN}{w^2} \Rightarrow N = N_0 e^{-t/\tau} \therefore -\frac{N_0}{\tau} e^{-t/\tau} = -\frac{8DN_0 e^{-t/\tau}}{w^2}$

$\therefore$ Time constant $\tau = \frac{w^2}{8D}$ so, $t_{\text{rise}} = 2.2\tau$ and $f_{-3dB} = \frac{1}{2\pi\tau}$

$$\therefore \tau = \frac{(250 \times 10^{-6})^2}{8 \times 4.14 \times 10^{-3}} = 1.89 \mu\text{s} \quad \therefore f_{-3dB} = B = 84.3 \text{ kHz}$$

Thermal noise = $0.31 \mu\text{V}_{\text{rms}} \approx 0.38 \mu\text{T} \approx 8.2 \mu\text{A}$
 equiv. current

[25%]

Q1 Ultrasonic level sensing

A popular question which was well answered by most. Most candidates made a good attempt at calculating the signal levels, which were quite complex in the latter stages with multiple media and some did not take correct account of the beam angle. The pulse-echo transit times were nearly always correct however.

Q2 MEMS fabrication, device capacitance & strain gauges

A quite well-answered question on MEMS fabrication (surface micro-machining), with most candidates knowing the main steps, however process details were often missing. The estimate of device capacitance often only included one side, although the sensing voltage feedback was generally quite well understood. The first part on strain gauge amplification attracted a variety of feasible circuits.

Q3 Optical smoke detector and pyrometer system

A very popular question, which was well answered in most cases and with a few model answers. Noise levels were generally well estimated using the amplifier bandwidth determined by the feedback RC network. The pyrometer analysis was generally correctly done, including the first part on detector thermal rating, although the algebra sometimes went awry.

Q4 Hall effect current sensors

This was a popular question, which attracted some high quality attempts - although some candidates failed to calculate the flux density and Hall sensor responsivity correctly. Many recalled the Hall effect sensor equations – or at least remembered the end result to use for the bandwidth estimate. The inductance calculation was not correct in a number of cases though.