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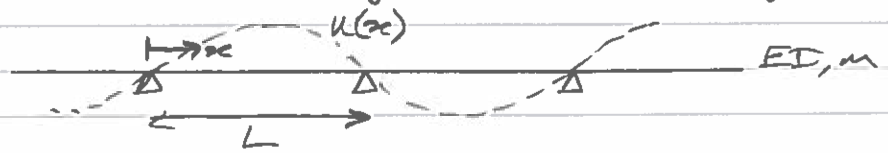
Part IIB 4D6: Dynamics in Civil Engineering

(1)



a) The assumption of pinned supports suggests a simple sinusoidal form to the assumed mode:

$$\bar{u} = \sin\frac{\pi x}{L} \quad ; \quad \bar{u}' = \frac{\pi}{L}\cos\frac{\pi x}{L} \ , \quad \bar{u}'' = \frac{-\pi^2}{L^2}\sin\frac{\pi x}{L} \qquad [2]$$

$$M_{eq} = \int_0^L m\bar{u}^2\,dx = \int_0^L m\sin^2\frac{\pi x}{L}\,dx$$ (integrating over one sleeper spacing)

$$= \frac{m}{2}\int_0^L 1 - \cos\frac{2\pi x}{L}\,dx = \frac{m}{2}\left[x - \frac{L}{2\pi}\sin\frac{2\pi x}{L}\right]_0^L = \frac{mL}{2} \qquad [2]$$

$$K_{eq} = \int_0^L EI\left(\frac{d^2\bar{u}}{dx^2}\right)^2 dx = EI\frac{\pi^4}{L^4}\int_0^L \sin^2\frac{\pi x}{L} = EI\frac{\pi^4}{L^4}\cdot\frac{L}{2} = \frac{\pi^4 EI}{2L^3} \qquad [2]$$

$$\Rightarrow \omega = \sqrt{\frac{K_{eq}}{M_{eq}}} = \sqrt{\frac{\pi^4 EI}{\cancel{2}L^3}\cdot\frac{\cancel{2}}{mL}} = \frac{\pi^2}{L^2}\sqrt{\frac{EI}{m}}$$

$$= \frac{\pi^2}{(0.6)^2}\sqrt{\frac{6.42\times10^6}{60}} = 8968 \text{ rad/s} \equiv 1427 \text{ Hz} \qquad [2]$$

b) Worst possible location for wheel impact is mid-span between sleepers, ie. at the antinode.

$$\Rightarrow F_{eq} = F = 400\text{ kN}. \qquad [2]$$

$$U_{static} = \frac{F_{eq}}{K_{eq}} \quad ; \quad K_{eq} = \frac{\pi^4\times6.42\times10^6}{2(0.6)^3} = 1448 \text{ MN/m}$$

$$= \frac{400\times10^3}{1448\times10^6} = 0.28 \text{ mm}. \qquad [2]$$

$$t_d = 2\text{ms} \quad ; \quad T = \frac{1}{f} = \frac{1}{1427} = 0.70\text{ms} \Rightarrow \frac{t_d}{T} = \frac{2}{0.7} = 2.9$$

$$\Rightarrow DAF \simeq 1.15$$

$$\therefore U_{dyn} = 1.15\times0.28 = 0.32\text{ mm} \qquad [2]$$

In practice, this SDOF model is limited due to the 'infinite' nature of the rail. The calculation in part a). does indeed estimate the 'pinned-pinned' natural frequency, which arises due to a

(1) continued

standing wave in the rail. However, the response calculated in part b). is limited due to:

- propagation of vibration along the rail (radiation damping)
- more complex behaviour arising due to the infinite, periodic nature of the structure
- excitation from other train wheels
- the effects of the moving load.

c). i). Rubber rail pads introduced at the supports would not influence the pinned-pinned mode if they behaved as simple linear springs, since they are located at nodal points (no vertical deflection):

However, the finite width of the pads may introduce rotational stiffness, which would tend to increase the natural frequency, due to the overall stiffness increase:

ii). Any increase in stiffness would tend to reduce the static deflection but the final dynamic deflection depends on both U_{stat} and the DAF associated with the new natural frequency.

SPT
28/5/16.

② a) Assumptions : $S_a = \omega^2 S_d$

$S_v = \omega S_d \approx \dot{v}_{max}$

↳ Assumes : Low damping, structure elastic

↳ Inaccurate for low frequencies.

b) Ductility : - provides more gradual collapse mechanism (preferred)

- allow more cost effective design (reduced base shear)
- allows load to redistribute to take advantage of redundancy
- lengthens the period which can reduce excitation
- provides energy dissipation.

② (c)

$$k_1 = \left(\frac{1}{2}\frac{EA_1}{L_b}\right)\overset{\swarrow \text{ braces}}{2} = \frac{EA_1}{L_b} = 2k_2$$

$$k_2 = \left(\frac{1}{2}\frac{EA_2}{L_b}\right) 2 = \frac{EA_2}{L_b} = \frac{210\,\text{GPa}}{\sqrt{4^2+4^2}}\left(100\ \text{mm}^2\right) = 3.7\times10^6\ \text{N/m}$$

$$m_1 = 10{,}000\ \text{kg}$$

$$m_2 = 20{,}000\ \text{kg} = 2m_1$$

$$\underline{k} - \omega^2 \underline{m} = k_2 \begin{bmatrix} 3 & -1 \\ -1 & 1 \end{bmatrix} - \omega^2 m_1 \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\lambda = \frac{\omega^2 m_1}{k_2} \rightarrow \det\begin{bmatrix} 3-\lambda & -1 \\ -1 & 1-2\lambda \end{bmatrix} = 0$$

$$(3-\lambda)(1-2\lambda) - 1 = 0 \rightarrow 2 - 7\lambda + 2\lambda^2 = 0 \rightarrow \underline{\lambda = 0.31,\ 3.19}$$

$$\omega = \sqrt{\lambda \frac{k_2}{m_1}} \rightarrow \omega = 10.7,\ 34.4\ \frac{\text{rad}}{\text{s}} \rightarrow \underline{\underline{f = 1.7,\ 5.5\ \text{Hz}}}$$

(d) First mode $\rightarrow$ $(3-0.31)\phi_1 + (-\phi_2) = 0 \rightarrow \phi = \begin{bmatrix} 0.37 \\ 1 \end{bmatrix}$

$$M_{eq} = (0.37)^2\,10{,}000 + (1)^2(20{,}000) = 21{,}370\ \text{kg}$$

$$\Gamma = \frac{0.37(10{,}000) + 1(20{,}000)}{M_{eq}} = 1.11$$

$$S_{a,\text{design}} = 2.5(0.2g) = 0.5g = 4.9\ \text{m/s}^2$$

$$S_{d,\text{design}} = \frac{S_a}{\omega^2} = \frac{4.9}{10.7^2} = 0.043\ \text{m}$$

② (d) (cont.)

$$u_{max} = \begin{bmatrix} 0.37 \\ 1 \end{bmatrix} (1.11)(0.043) = \begin{bmatrix} 0.018 \\ 0.048 \end{bmatrix} m \rightarrow \underline{\underline{\text{Max drift} = 0.030\,m}}$$

$$\ddot{u}_{max} = \begin{bmatrix} 0.37 \\ 1 \end{bmatrix} (1.11)(4.9) = \begin{bmatrix} 2.0 \\ 5.4 \end{bmatrix} m/s^2$$

$$V_{base} = \Sigma ma = (10{,}000)(2.0) + 20{,}000\,(5.4)$$
$$= 20{,}000 + 108{,}000 = 128{,}000\ N$$

bottom storey $\Big\}$ $F_{brace} = \dfrac{128000}{2}\sqrt{2} = 90{,}500\ N$

$$F_{yield} = \sigma_y A = 275 \times 10^6 \frac{(200\ mm^2)}{1000^2} = 55{,}000\ N$$

$$\frac{F_{el}}{F_y} = \sqrt{2\mu - 1} \rightarrow \mu = \frac{\left(\frac{F_{el}}{F_y}\right)^2 + 1}{2} \rightarrow \mu = 1.85$$

Top storey $\Big\}$ $F_{brace} = \dfrac{108{,}000}{2}\sqrt{2} = 76{,}400\ N$

$F_{yield} = \dfrac{55{,}000}{2} = 27{,}500\ N$

$\Big\}$ $\underline{\underline{\mu = 4.35}}$

Q3) a) Consider the water tank is empty.

Volume of Concrete = $(3^3 - 2.5^3) = 11.375\ m^3$.

$\therefore$ Mass of water tank = $2400 \times 11.375 = 27300\ kg$

Flexural stiffness = $EI = 30 \times 10^6\ Nm^2$.

$$k = \frac{3EI}{L^3} = \frac{3 \times 30 \times 10^6}{11^3} = 67618.33\ N/m$$

[15 %]

$$\therefore f_n = \frac{1}{2\pi}\sqrt{k/m} = \underline{0.25}\ Hz$$

b) water tank full

$$k = \frac{3 \times 30 \times 10^6}{11.5^3} = 59176.46\ N/m$$

$$m = 27300 + 2.5^3 \times 1000 = 42925\ kg$$

[15 %]

$$\therefore f_n = \frac{1}{2\pi}\sqrt{K/m} = \underline{0.187}\ Hz$$

c) Consider water tank empty case

Bearing pressure under raft = $\frac{50950 \times 9.81}{2^2} = 124.95\ kPa$

unit weight of soil = $14.3\ kN/m^3$.

$\sigma_v' = \sigma_v = 124.95 + 14.3 = 139.25\ kPa$

From Data sheets: $G_{max} = 100 \frac{[3-e]^2}{1+e} (p')^{0.5}$

void ratio $e = 0.75$ $K_0 = \frac{V}{1-V} = \frac{0.3}{1-0.3} = 0.428$

$p' = \left(\frac{1+2K_0}{3}\right)\sigma_v' = 0.619\,\sigma_v' = 86.20\ kPa$.

$$\therefore G_{max} = 100 \frac{[3-0.75]^2}{1.75} \times \left(\frac{86.20}{1000}\right)^{1/2} = \underline{84.935\ MPa}$$

$l/b = 1$ $e/b = 0.5$

$$K_r = \frac{Gb^3}{1-V}[4]\left[1 + \frac{0.5}{1} + \frac{1.6}{1.35}(0.5)^2\right] = \frac{7.185\,Gb^3}{0.7} = 10.26\,Gb^3$$

$\therefore$ Rotational Stiffness of sand = $10.26 \times 1^3 \times 84.935 = \underline{871.82\ MNm/rad}$

3 c) contd. Consider water tank is full case.

$$\text{Bearing pressure} = \frac{66\,575 \times 9.81}{4} = 163.275 \text{ kPa}$$

$$\sigma_v' = 1 \times 14.3 * 163.275 = 177.575 \text{ MPa}$$

$$p' = \left(1 + \frac{2K_0}{3}\right)\sigma_v' = 109.93 \text{ kPa}$$

$$G_{max} = 100 \frac{[3-e]^2}{1+e} (p')^{0.5} = \underline{\underline{95.91 \text{ MPa}}}.$$

$$K_r = 10.26 \times 95.91 = \underline{\underline{984.44}} \text{ MN-m/rad}$$ [25%]

d) A simple discrete model for rocking can be

Consider tank empty case:

From part a) lumped mass = 27300

$$I_{struct} = ma^2 = 27300\,[1 + 10 + 1.5]^2 = 4265625 \text{ kg·m}^2$$

$$I_{soil} = 18 \times 10^6 \text{ kg-m}^2$$

$$I = 22.26 \times 10^6 \text{ kg-m}^2$$

$$\therefore \omega_n = \sqrt{\frac{K_r}{I}} = \sqrt{\frac{871.82}{22.26 \times 10^6}} = \underline{\underline{0.9959}} \approx \underline{\underline{1.0 \text{ Hz}}}$$

Tank full case: From part b) lumped mass = 42925

$$I_{str} = 42925\,(1.5 + 10 + 1.5)^2 = 7.2543 \times 10^6$$

$$I_{soil} = 20 \times 10^6 \text{ kg m}^2$$

$$I = 27.2543 \times 10^6$$ [25%]

$$\therefore \omega_n = \sqrt{\frac{984.44 \times 10^6}{27.2543 \times 10^6}} = \underline{\underline{0.956 \text{ Hz}}}$$

e) By not considering the foundation soil, the natural frequency of the water tank may be estimated to be quite low and well below the frequency range of most earthquakes (1~5 Hz). This may lead us to believe that the structure is safe form earthquakes. However by considering rocking mode, the natural frequency of water tank in both cases is quite close to 1 Hz. This can lead to resonance and hence severe damage to the water tank. It is therefore very important to considering the soil-structure system together and include SSI effects. [20%]

Simply supported beam.

4 a) Assume $\bar{u} = \sin\frac{\pi x}{L}$

$$M_{eq} = \int_0^L m\, a^2 \sin^2\frac{\pi x}{L}\, dx = m a^2 \cdot \frac{L}{2}$$

$$K_{eq} = \int_0^L EI \left(\frac{d^2 u}{dx^2}\right)^2 dx = EI\left(\frac{\pi^2}{L^2}\right)^2 \underbrace{\int_0^L a^2 \sin^2\frac{\pi x}{L}\, dx}_{L/2} = \frac{\pi^4 EI}{2L^3}$$

$$\omega = \sqrt{\frac{K}{m}} = \sqrt{\frac{\pi^4 EI \cancel{2}}{\cancel{2}L^3\, mL}} = \frac{\pi^2}{L^2}\sqrt{\frac{EI}{m}}$$

$D = 193.7$, $t = 5.0$ mm

$E = 210 \times 10^9$ N/m^2

$I = 1320 \times 10^{-8}$ m^4

$m = 24.6 \times 10^{-4}$ m^2 $\times$ 7840 kg/m^3 = 23.2 kg/m (23.3 kg/m Struct. D. Book)

$$\omega = \frac{\pi^2}{8^2}\sqrt{\frac{(210\times10^9)(1320\times10^{-8})}{23.2}} = 53.3 \text{ Radians/sec}$$

$f = \frac{\omega}{2\pi} = 8.48$ Hz

$$St = \frac{nD}{U} \rightarrow U = \frac{nD}{St} = \frac{(8.48\, \text{Hz})(0.1937\, \text{m})}{0.2}$$

$= \cancel{12.8}$ m/s 8.2 m/s.

b) $$S_c = \frac{2\,\delta\, m}{\rho D^2} = \frac{2(2\pi(0.005))(23.3)}{(1.25)(0.1937)^2} = 31.2.$$

$$\frac{y_{max}}{D} = \frac{1.5}{S_c} \Rightarrow y_{max} = \left(\frac{1.5}{31.2}\right) \times (0.1937) = 0.009\ \text{m} = 9.3\ \text{mm}$$

c) Increase wall thickness by factor λ, say

then mass per unit length increases by factor $\approx \lambda$

stiffness EI increases by factor $\approx \lambda$

$\therefore$ frequency $\approx$ unchanged

$\therefore$ critical windspeed unchanged

However, Scruton number increases by factor λ

$\therefore$ deflections decrease by factor $1/\lambda$

So fatigue life is improved, because of lower stress range.

d) – Increase wall thickness, as per c).

– Increase the damping – eg. TMD's

– Helical strakes or shrouds (but this increases "static" drag).

e). Underwater –

i) need to use "added mass"

ii) Would need to look out for in-line shedding 

(as per Immingham jetty).

Q1

All candidates attempted Q1, and generally produced respectable solutions. The majority of candidates correctly estimated the fundamental natural frequency via the equivalent mass and stiffness of a half-sine (or occasionally quadratic) mode shape. The majority went on to recognise that the worst loading point was mid-span and correctly estimate the response. Most marks were lost for failing to offer plausible comments on the short comings of using an equivalent SDOF model for what amounts to an infinite system (a continuous beam on periodic supports). Approximately half the candidates offered correct, structured arguments for part (c).

Q2

About half the candidates correctly stated the assumptions made to create a tripartite response spectrum, while a majority did well explaining the benefits of ductility. Most candidates followed an appropriate procedure to calculate the natural frequency of the frame and the maximum inter-storey drift, while relatively few calculated the stiffness correctly.

Q3

This was a very popular question on the soil-structure interaction aspects, with almost all the candidates attempting it. By and large, almost all the candidates could calculate the natural frequency of a fixed-base water tank when either empty or full. In the later parts of the question they were required to calculate the stiffness of the foundation soil. Again, this was done well but a few candidates did not include the vertical stress due to the water tank. The final part required them to calculate the natural frequency of the soil-structure system by considering a SDOF. However, many candidates tried to set up a 2DOF system and worked out the two natural frequencies. Although this took a bit longer to calculate, it still yields the same answers, and therefore the candidates were not penalised for taking this approach. Almost all the candidates could identify the importance of considering SSI effects in this problem.

Q4

This question about vortex shedding was attempted by only six candidates, most of whom were able to successfully predict the resonance wind speed and amplitude, and comment sensibly on how these would be affected by differing conditions.

JPT/MSPG/MJD/FAM