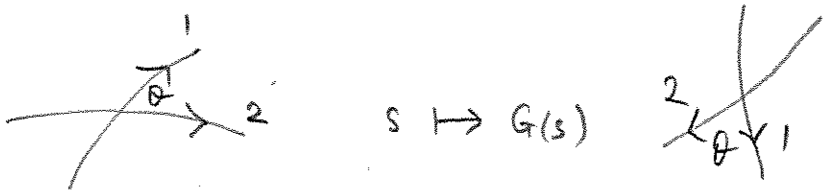
4F1 solutions (2017)

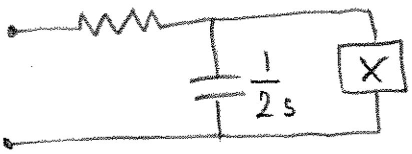
1 (a) A mapping $s \mapsto G(s)$ is conformal if $G(s)$ is analytic and $\frac{d}{ds} G(s) \neq 0$. A conformal mapping preserves angle and sense:



Usefulness: (1) justification of rule for breakaway points in the root-locus diagram,
(2) informal proof of Nyquist stability criterion,
(3) estimate of location of dominant poles in a feedback system.

(b)(i)

$X \rightarrow$



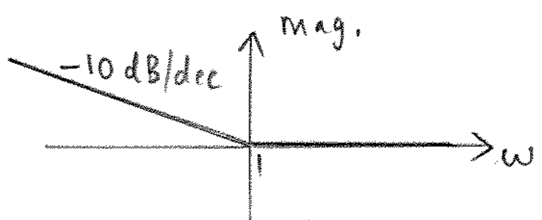
Impedance of parallel connection $= (2s + X^{-1})^{-1}$

(ii)
$$1 = (X-2)(2s+X^{-1})$$
$$= 2sX + 1 - 4s - 2X^{-1}$$
$$\Rightarrow \quad 2sX^2 - 4sX - 2 = 0$$
$$\Rightarrow \quad X = \frac{4s \pm \sqrt{16s^2 + 16s}}{4s} = 1 \pm \sqrt{1 + \frac{1}{s}}$$
$$\Rightarrow \quad Z = \pm \sqrt{\frac{s+1}{s}}$$

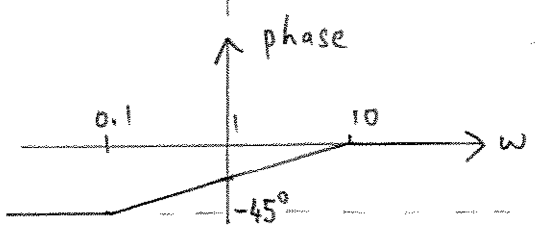
For large s circuit behaves like a $1\,\Omega$ resistor, hence + sign is the correct one.

(iii)

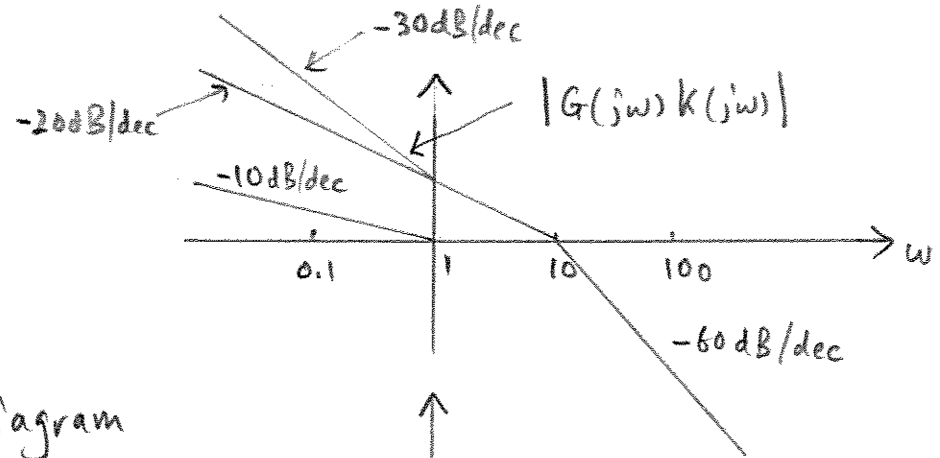
$$Z(j\omega) = \sqrt{\frac{1+j\omega}{j\omega}}$$



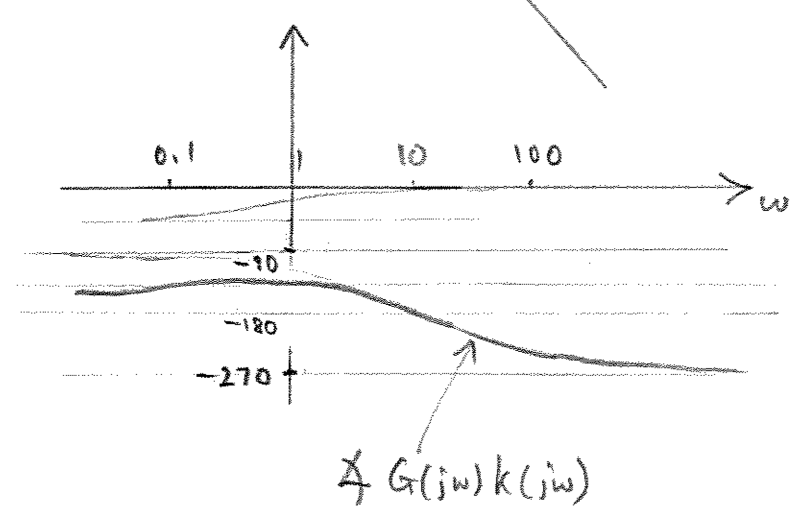
straight-line approximation

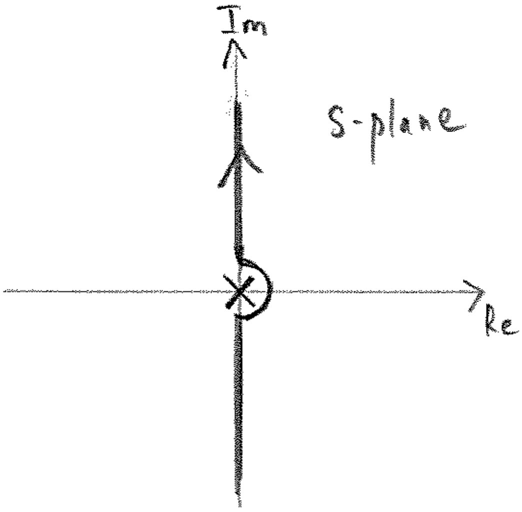


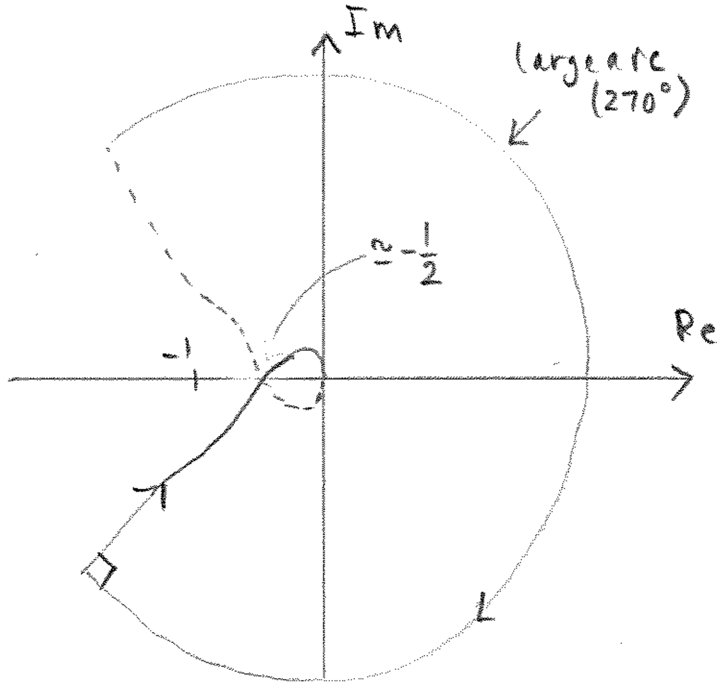
1 (iv)



Bode diagram



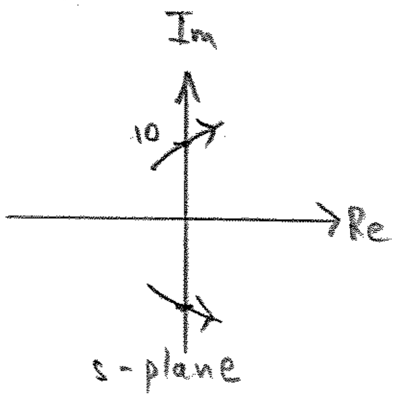




$$\measuredangle G(j10)\,k(j10) \simeq -180^\circ$$

$$|G(j10)\,k(j10)| \simeq \frac{1}{2}$$

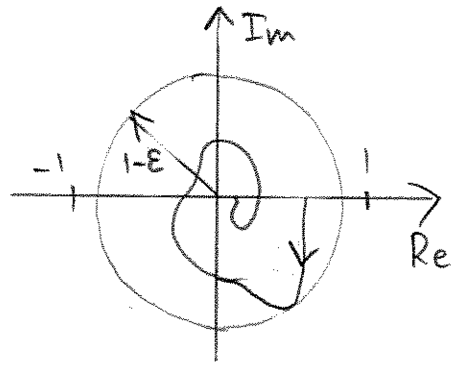
Locus of solutions mapped by $G(s)K(s)$ to ≈ -0.5



(v) Closed-loop stable for $0 < k < 2$
2 RHP poles $2 < k < \infty$
1 RHP pole $k < 0$

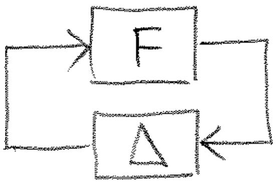
(vi) A pair of dominant poles moves from the LHP to RHP with frequency increasing through 10 rad/sec as k increases through 2.

2(a)(i)



Nyquist diagram of $F(s)\Delta(s)$ (stable) lies inside a circle of radius $1-\varepsilon$, and hence cannot encircle ± 1 point. Feeback system is stable by Nyquist stability criterion.

(ii)



A closed-loop transfer function:

$$\frac{1}{1 - F\Delta}$$

Let $c = \dfrac{\pm 1}{|F(j\omega_0)|}$ and $-\pi < \angle \left[\dfrac{1 - j\omega_0 T}{1 + j\omega_0 T}\right] \leq 0$

for $T \geq 0$. Hence by choice of $\pm$ sign and $T \geq 0$ we can always ensure that

$$F(j\omega_0)\,\Delta(j\omega_0) = 1$$

which makes the loop unstable. Note that

$$|\Delta(j\omega)| = \frac{1}{|F(j\omega_0)|} = f(\omega_0) \leq f(\omega)$$

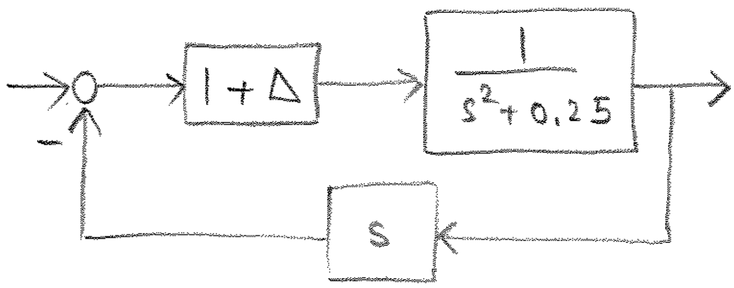
for all ω, so the condition of (a)(i) is satisfied.

(b) Bookwork – block diagram rearrangement.
Necessary and sufficient condition:

$$\left| \left(\frac{G_0 k}{1 + G_0 k}\right)(j\omega) \right| < \frac{1}{h(\omega)}$$

for all ω.

(c)(i)



(c) (ii)

$$\frac{G_0 K}{1 + G_0 K} = \frac{\frac{s}{s^2+0.25}}{1 + \frac{s}{s^2+0.25}} = \frac{s}{s^2 + s + 0.25}$$

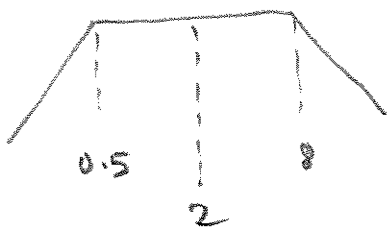
Need

$$\delta_0 \left| \left(\frac{j\omega + 0.5}{j\omega + 0.8} \right) \left(\frac{j\omega}{(j\omega + 0.5)^2} \right) \right| \leqslant 1 \qquad (1)$$

for all ω.

$$(1) \Leftrightarrow \quad \delta_0 \frac{\omega}{|(j\omega + 0.5)(j\omega + 8)|} \leqslant 1$$

Bode magnitude:



geometric mean → maximum

$$\text{Hence, } (1) \Leftrightarrow \quad \delta_0 \frac{2}{|(j2+0.5)(j2+8)|} \leqslant 1$$

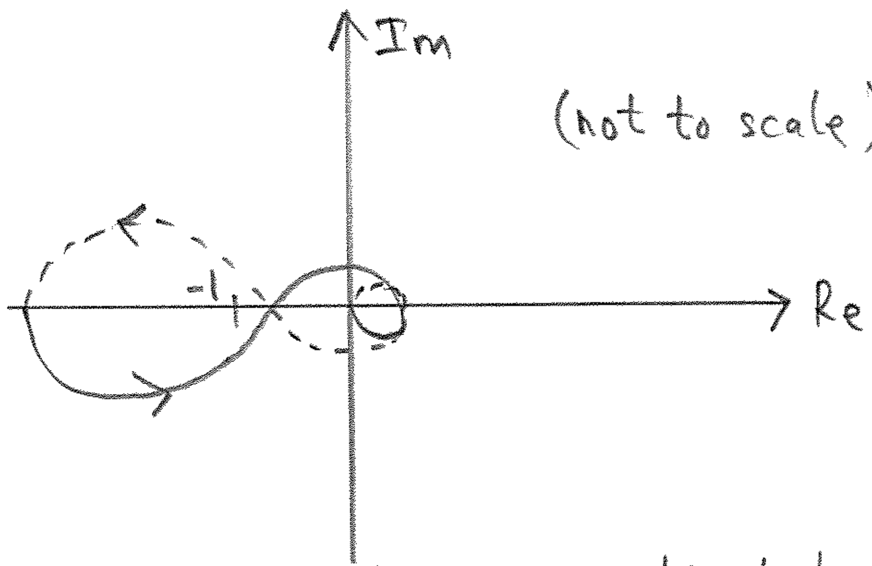
$$\Leftrightarrow \quad \frac{2\delta_0}{0.5(1+4^2)\,2} = \frac{2\delta_0}{17} \leqslant 1$$

$$\Leftrightarrow \quad \delta_0 \leqslant 8.5$$

Choose $\delta_0 = 8.5$.

(iii) Although (b) is a necessary and sufficient condition the form of $\Delta(s)$ in (c) is restricted, so result <u>might</u> be conservative.

3(a)(i)



(ii) Nyquist diagram shows one anti-clockwise encirclement of −1 point. Since feedback loop is stable with k=1 there must be one pole of G(s) in the RHP.

(b)(ii) Phase excess rises (due to RHP pole) and then decreases to $-180°$ indicating two RHP zeros. Estimated location:

$s = 10$ RHP pole
$s = 200$ 2 RHP zeros

[Actual transfer function

$$G(s) = \frac{10(s-100-100j)(s-100+100j)}{(s+60)(s+100)(s-10)}$$]

(iii) Crossover frequency couldn't be much lower than 10 rad/sec or much higher than 200 rad/sec.

(c)(i)

$$20\log_{10} 0.72 = -2.85 \text{ dB}$$
$$20\log_{10} 1.8 = 5.11 \text{ dB}$$
$$2\tan^{-1} 1.8 - 90 = 31.89°$$

Gain reduced by 2.85 dB at $w = 20$ rad/sec so $w = 20$ is new gain crossover frequency.

(c)(i) cont.

Gain reduced by around 8 dB at $s = 0$.

Gain increased by around 2 dB at high frequency.

Phase at $w=20$: $180 + 20 + 32$

$\Rightarrow$ phase margin $= 52^\circ$

(ii) Gain at $s=0 \simeq 11-8 = 3$ dB.

So gain reduction of 3dB gives instability.

Phase $= 180^\circ$ at $w \simeq 50$ rad/sec, where gain $\simeq -10$dB. So gain increase of 10 dB gives instability.

$\Rightarrow$ gain margin $=$ 3 dB

(iii) Try a lag compensator

$$K_2(s) = \frac{s+8}{s+5.5}$$

Increases gain at $w=0$ by 3.25 dB

Introduces phase lag at $w=20$ equal to:

$$\tan^{-1}\frac{20}{8} - \tan^{-1}\frac{20}{5.5} = -6.42^\circ$$

which should preserve phase margin $> 45^\circ$

Gain should be $>$ 6dB at $w=0$ and < -10dB at $w=50$ so gain margin should be $>$ 6 dB.

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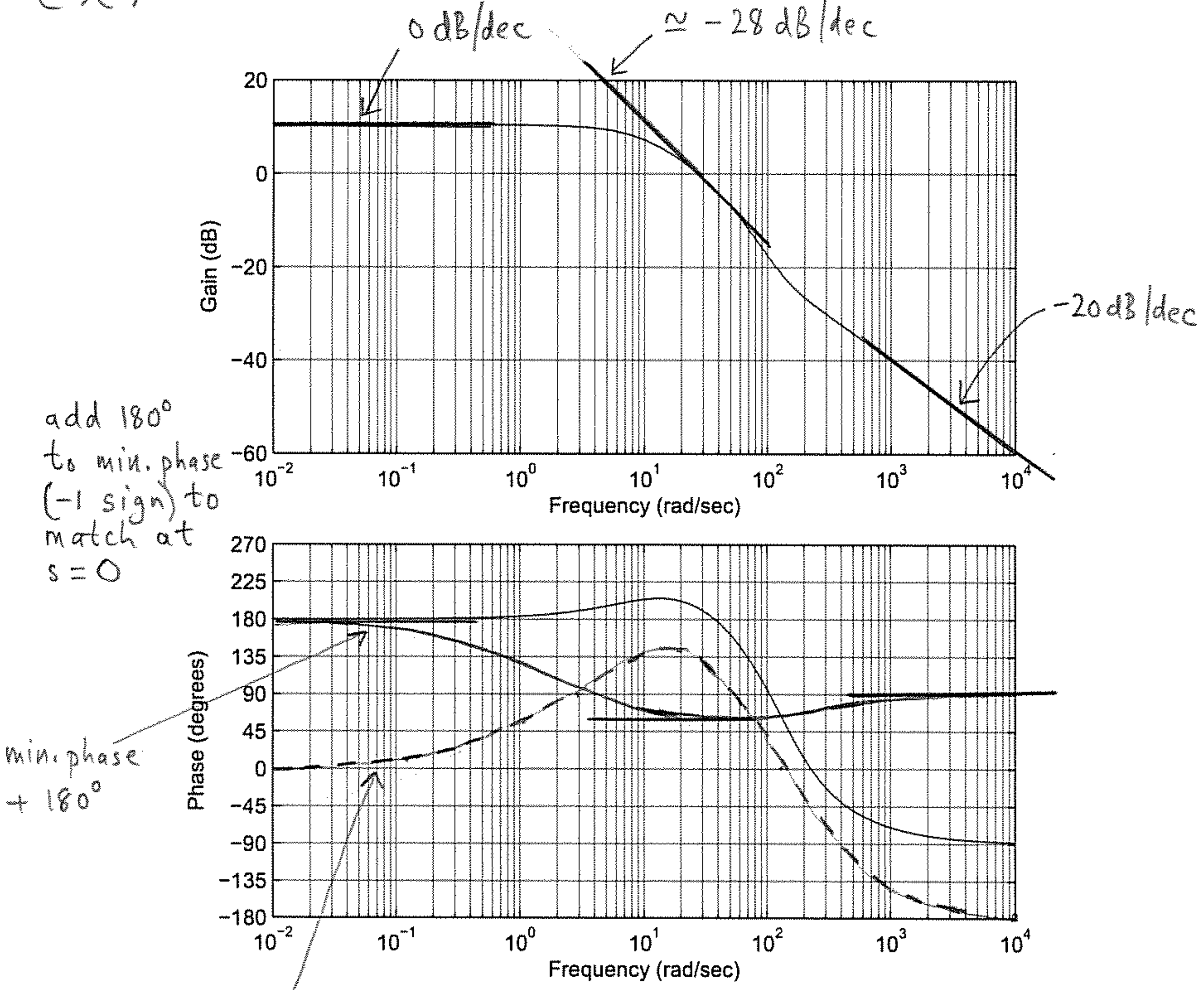
(b)(i)



Extra copy of Fig. 2: Bode diagram for Question 3.

phase excess

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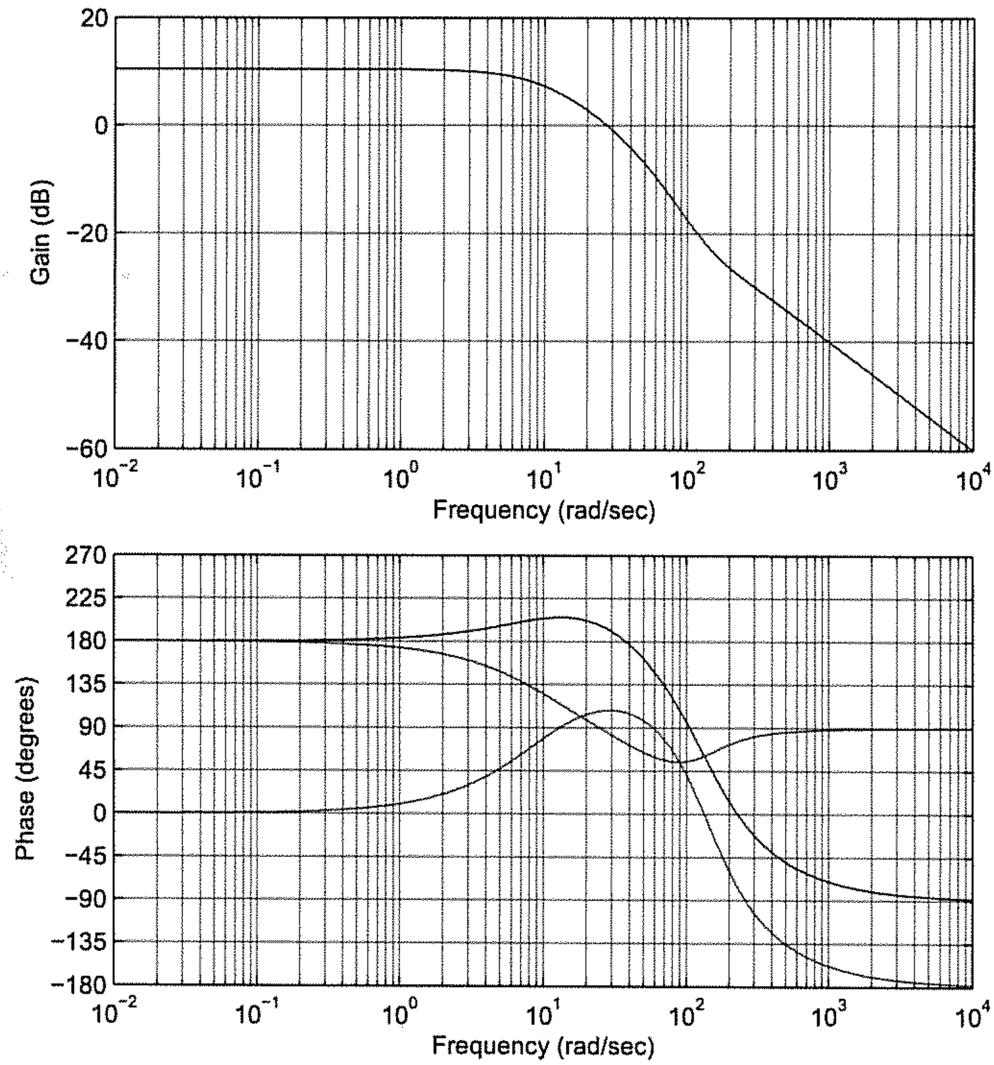
(b)(i) Accurate computer plot.



Extra copy of Fig. 2: Bode diagram for Question 3.

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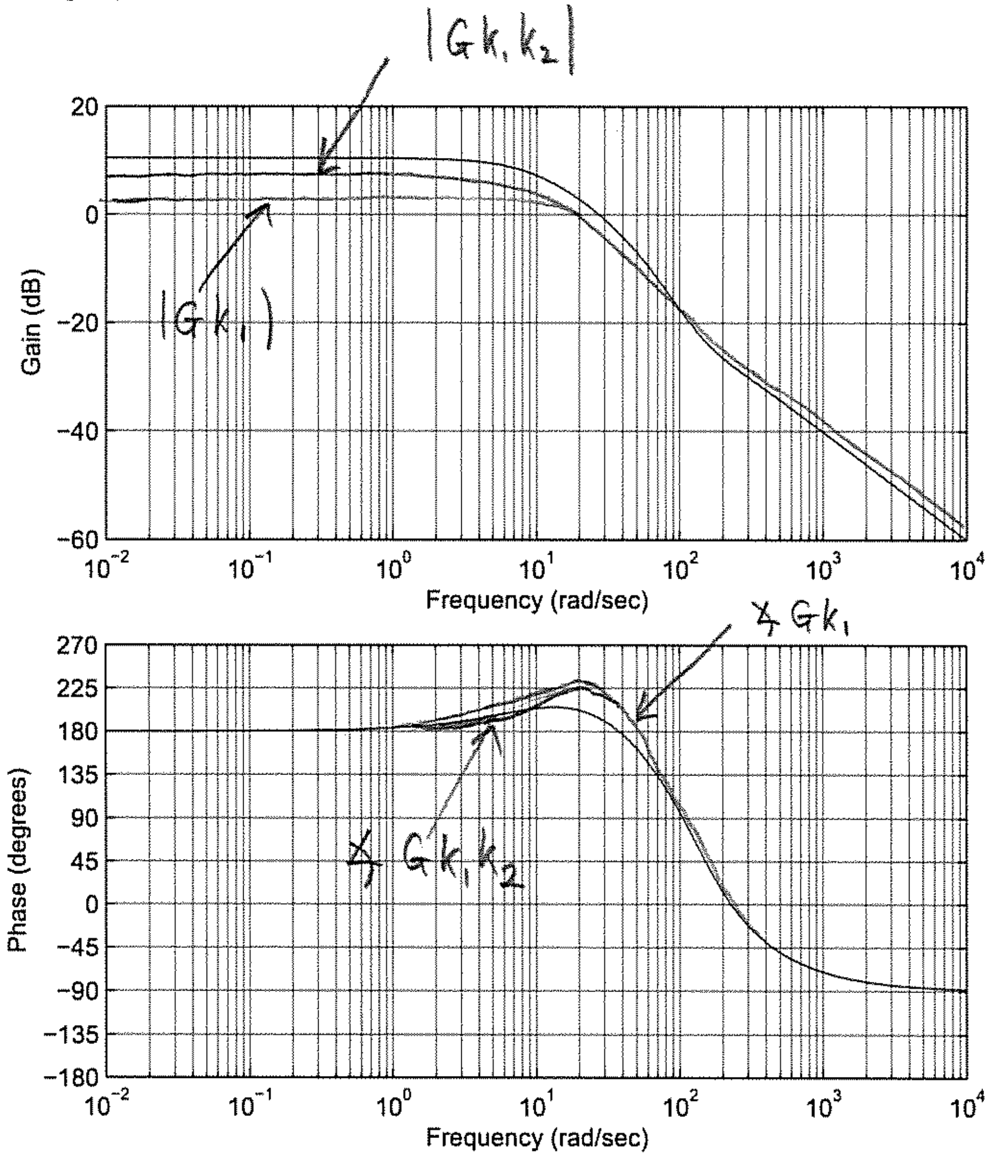
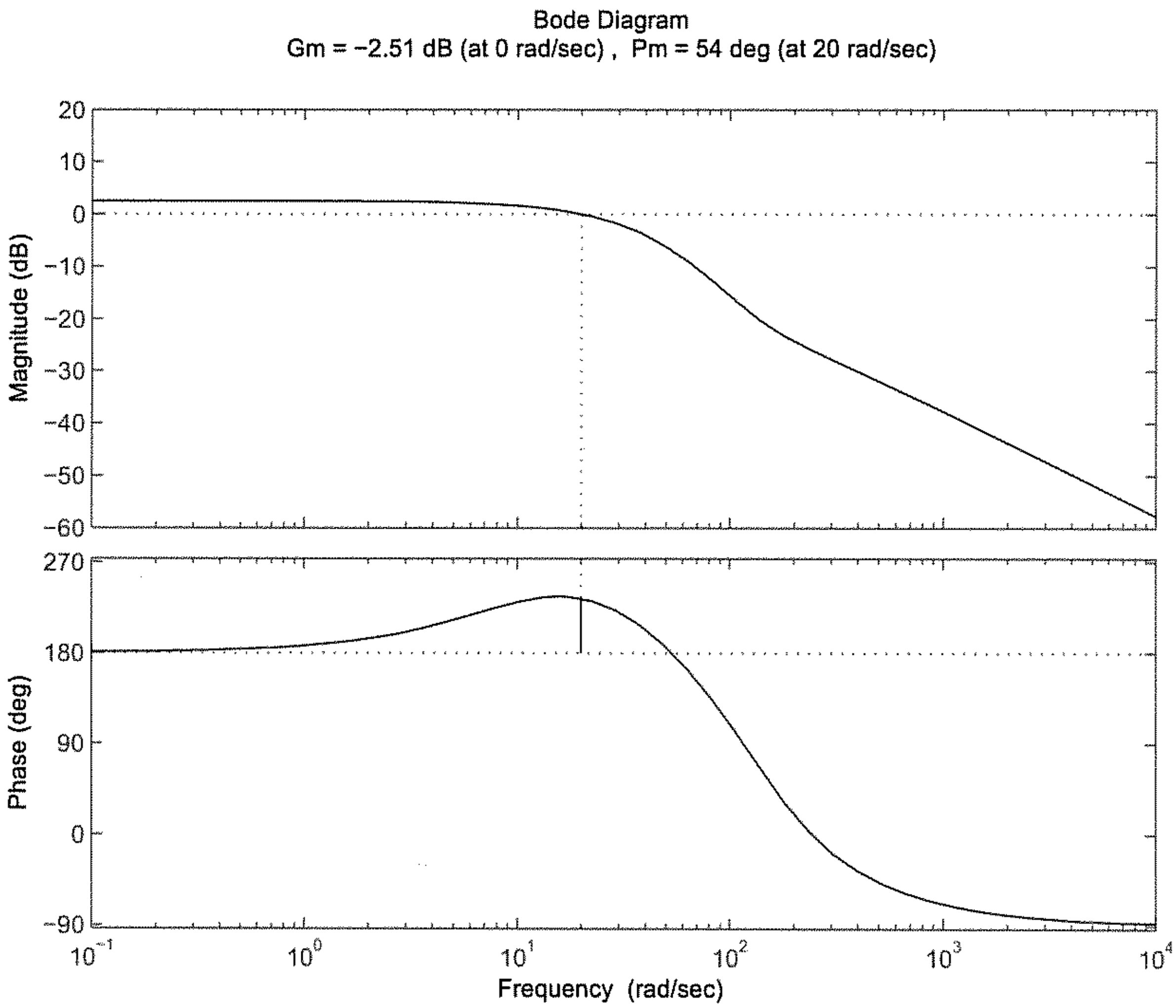
(c)(i) - (iii)



Extra copy of Fig. 2: Bode diagram for Question 3.

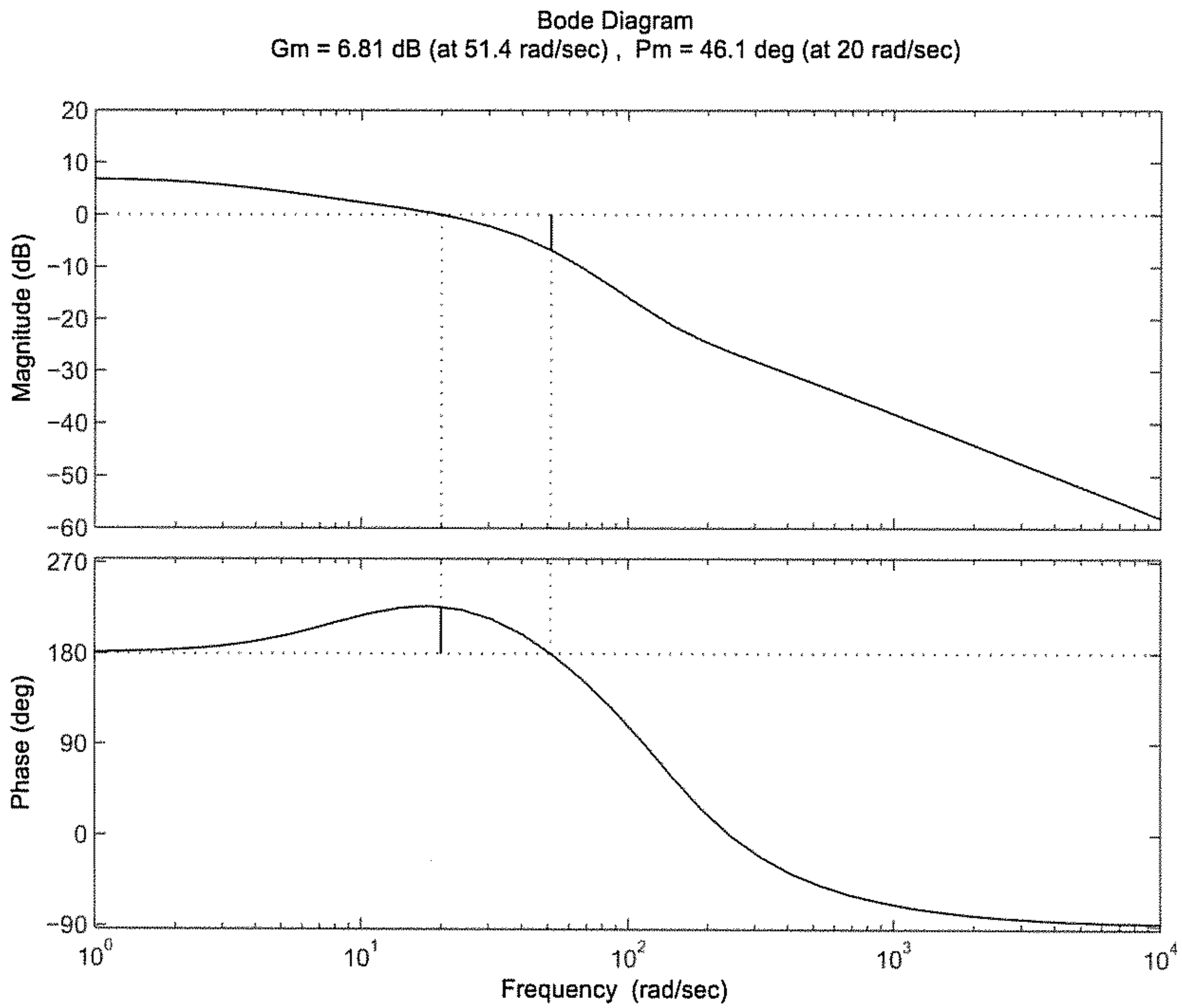
(c)(i) Accurate computer plot.



(cont.

(c) (iii) Accurate computer plot.

Version MCS/1



Actual $k_2(s) = 0.963 \dfrac{s+7}{s+4}$

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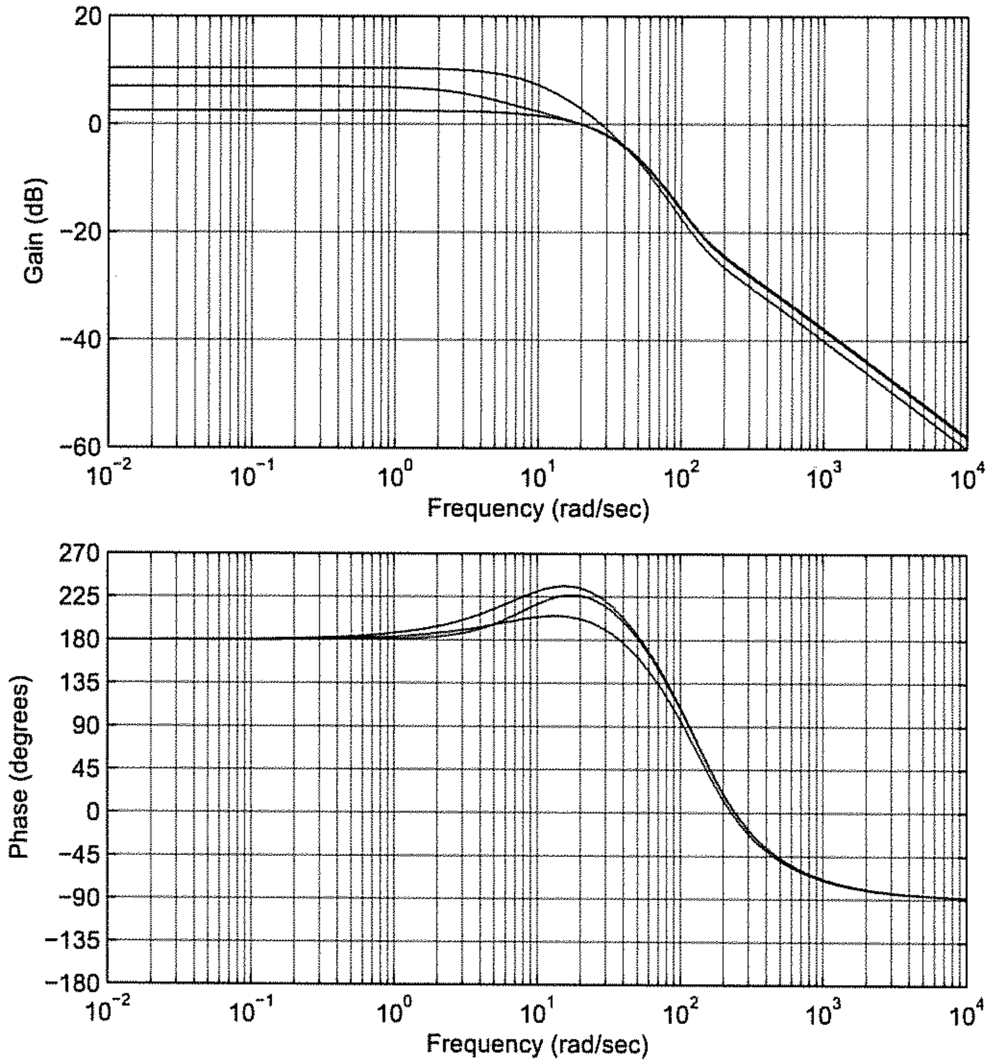
(c)(i)-(iii) Accurate computer plot



Extra copy of Fig. 2: Bode diagram for Question 3.

ENGINEERING TRIPOS PART IIB 2017
ASSESSOR'S REPORT, MODULE 4F1
Control System Design

Q1. Conformal mapping. Non-rational compensator.
10 attempts, mean 12.3/20 (61.5%), maximum 19, minimum 5.
An unpopular question. Candidates may have been put off by a compensator which was a square-root of a rational function. Apart from a couple of attempts which didn't get far, most candidates made good progress and were successful to apply the standard principles to a new situation.

Q2. Robustness and VTOL aircraft.
27 attempts, mean 11.63/20 (58.15%), maximum 18, minimum 5.
A popular question but with rather mixed performance from candidates. The bookwork was somewhat poorly done, especially part 2(a)(ii). A lot of candidates got bogged down with lengthy calculations on part 2(c)(ii) and failed to see that a simple Bode plot would readily lead to a solution.

Q3. Bode gain/phase and compensator design.
27 attempts, mean 14/20 (70%), maximum 19, minimum 9.
A popular question with many first class answers that showed a very good grasp of this material.