Q1)

(a)

(i) Assuming that all acceptors are ionized

$N_A = p = N_V e^{\left[\frac{-E_F}{K_B T}\right]}$=8x10^{24} exp [-0.06/(8.617x10^{-5}x298)]=7.7 x 10^{23} m^{-3}

(ii) From the Law of Mass Action:

$$n_i = \sqrt{N_C N_V} e^{\left(\frac{-E_g}{2K_B T}\right)}$$

Hence,

$N_C = n_i^2/N_V\, e^{\left(\frac{E_g}{K_B T}\right)}$= 10^{32}/(8x10^{24}) exp[1.1/(8.617x10^{-5}x298)]=5.02 x 10^{25} m^{-3}

From this, the density of donors can be found, again assuming that all are ionized:

N_D=n

$E_F - E_C = k_B\, T \ln\left(\frac{N_D}{N_C}\right)$=$8.617 \times 10^{-5} \times 298 \times \ln\left(\frac{7.7\times10^{23}}{5.02\text{x}10^{25}}\right) = -0.107\ eV$

(b) (i) 'onsider the p-type side of the junction. The Poisson equation states that

$$\nabla^2 V = \frac{-\rho}{\varepsilon_0 \varepsilon_r} = \frac{-eN_A}{\varepsilon_0 \varepsilon_r}$$

Given that V only varies in the x direction, then

$$\frac{\mathrm{d}^2 V}{\mathrm{d}x^2} = \frac{-eN_A}{\varepsilon_0 \varepsilon_r}$$

Integrating the equation gives the general solution,

$$\frac{\mathrm{d}V}{\mathrm{d}x} = \frac{-eN_A x}{\varepsilon_0 \varepsilon_r} + C$$

We know that at x=w_p the electric field is zero, so

$$C = \frac{eN_A w_p}{\varepsilon_0 \varepsilon_r}$$

and hence,

$$\frac{\mathrm{d}V}{\mathrm{d}x} = \frac{eN_A (w_p - x)}{\varepsilon_0 \varepsilon_r}$$

Integrating again gives the general solution,

$$V = \frac{eN_A}{\varepsilon_0 \varepsilon_r}\left(K + w_p x - \frac{x^2}{2}\right)$$

We can make $x = 0$ our arbitrary reference point, and set V at this point to be zero, so $K = 0$ and then the voltage on the p-side, V_p is simply the potential at $x = w_p$, so

$$V_p = \frac{eN_A w_p^2}{2\varepsilon_0 \varepsilon_r}$$

As the n-type side of the junction has an equal doping density, then the width of the junction on the n side must be the same, as must be the potential difference, so

$$V_p = \frac{eN_A w_p^2}{2\varepsilon_0 \varepsilon_r} = V_n = \frac{eN_D w_n^2}{2\varepsilon_0 \varepsilon_r} = \frac{V_0}{2}$$

Hence,

$$w = 2w_p = 2\left(\frac{\varepsilon_0 \varepsilon_r V_0}{eN_A}\right)^{1/2}$$

b(ii) For a pn junction

$eV_0 = E_{Fn} - E_{Fp} = E_C + (E_{fn} - E_C) - E_{Fp} = 1.1 + (-0.107) - 0.06 = 0.933$ eV

Hence

$$w = 2\left(\frac{12\times 8.854\times 10^{-12}\times 0.933}{1.602\times 10^{-19}\times 7.7\times 10^{23}}\right)^{0.5} = 56.7nm$$

Q2)

(a)

$$N = \int_0^{\infty} g(E)f(E)\,\mathrm{d}E = \int_0^{E_F} g(E)\,\mathrm{d}E$$

Substituting $g(E)\mathrm{d}E = \frac{\mathrm{V}}{2\pi^2\hbar^3}(2m)^{\frac{3}{2}}E^{\frac{1}{2}}\mathrm{d}E$ we get

N= $\frac{V}{2\pi^2\hbar^3}(2m)^{\frac{3}{2}}\int_0^{E_F} E^{\frac{1}{2}}\,\mathrm{d}E = \frac{V}{2\pi^2\hbar^3}(2m)^{\frac{3}{2}}\frac{2E_F^{\frac{3}{2}}}{3}$

Therefore:

$$E_F = \frac{\hbar^2}{2m}\left(3\pi^2\frac{N}{V}\right)^{\frac{2}{3}}$$

(b) Each atom contributes 2 free electrons:

$N/V = 2\text{x}3.92 \text{ x } 10^{28} = 7.84\text{x}10^{28}$ m^{-3}

Hence:

$E_F = \frac{\hbar^2}{2m}\left(3\pi^2\frac{N}{V}\right)^{\frac{2}{3}} = 6.69$ eV

(c)

$$E = \frac{\hbar^2|\boldsymbol{k}|^2}{2m}$$

$k = \frac{\sqrt{2mE}}{\hbar} = 1.324 \text{ x } 10^{10}$ m^{-1}

$\lambda = \frac{2\pi}{k} = 4.75 \text{ x } 10^{-10}$ m

P= $\hbar$k = mv

v= $\frac{\hbar \mathrm{k}}{\mathrm{m}} = 1.53 \text{ x } 10^{6}$ m/s

Problem 3a

(i) $\because \delta n_p(x) > n_{po}$ and $\delta p_n(x) > p_{no}$ $\Rightarrow$ diode is forward biased

(ii) The equilibrium minority carrier concentration:

$$n_{po} < p_{no}$$

$$\Rightarrow \frac{n_i^2}{p_{po}} < \frac{n_i^2}{n_{no}} \Rightarrow \boxed{n_{no} < p_{po}}$$

↳ majority carrier contration on the n-side is smaller than on the p-side.

(iii) Emission of light from a p-n junction corresponds to energy loss in the form of electromagnetic radiation. Energy loss occurs due to recombination of electrons and hole in the junction or depletion region. Hence, the diode has to be forward biased for the carriers to cross the depletion region and recombine to emit light.

(iv) Two ideal diodes at $T = 300K$

<u>Diode 1:</u> $E_{g1} = 0.525 eV$

$I_1 = 10 mA$ at $V_{a1} = 0.255 V$

<u>Diode 2:</u> $E_{g2} = ??$

$I_2 = 10 \mu A$ at $V_{a2} = 0.32 V$

The current in a FB diode is given by:

$$I = A \cdot J_s \cdot \left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]$$

$$\Rightarrow I = A \cdot \left[\frac{eD_p \cdot p_{no}}{L_p} + \frac{eD_n \cdot n_{po}}{L_n}\right] \cdot \left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]$$

$$\Rightarrow I = A \cdot e \cdot n_i^2 \left[\frac{D_p}{L_p \cdot N_D} + \frac{D_n}{L_n N_A}\right] \cdot \left[\exp\left(\frac{eV_a}{kT}\right) - 1\right] \quad \text{— (1)}$$

$\because$ the two diodes have exact same physical parameters except bandgap $\Rightarrow$ the only parameter in eqn (1) dependent on E_g is n_i^2 $\quad \because n_i^2 \propto \exp\left\{\frac{-E_g}{kT}\right\}$

Hence we can write,

$$I_1 \propto n_{i1}^2 \cdot \left[\exp\left\{\frac{eV_{a1}}{kT}\right\} - 1\right] \propto \exp\left\{\frac{-E_{g1}}{kT}\right\}\left[\exp\left(\frac{eV_{a1}}{kT}\right) - 1\right]$$

and $$I_2 \propto n_{i2}^2 \left[\exp\left(\frac{eV_{a2}}{kT}\right) - 1\right] \propto \exp\left\{\frac{-E_{g2}}{kT}\right\}\left[\exp\left(\frac{eV_{a2}}{kT}\right) - 1\right]$$

$$\therefore \frac{I_1}{I_2} = \exp\left\{\frac{E_{g2} - E_{g1}}{kT}\right\} \cdot \frac{\left[\exp\left(\frac{eV_{a1}}{kT}\right) - 1\right]}{\left[\exp\left(\frac{eV_{a2}}{kT}\right) - 1\right]}$$

$$\Rightarrow \exp\left(\frac{E_{g2} - E_{g1}}{kT}\right) = \frac{I_1}{I_2} \cdot \frac{\left[\exp\left(\frac{eV_{a2}}{kT}\right) - 1\right]}{\left[\exp\left(\frac{eV_{a1}}{kT}\right) - 1\right]}$$

$$\Rightarrow \left(E_{g2}-E_{g1}\right) = KT \ln\left\{ \frac{I_1}{I_2} \cdot \frac{\left[\exp\left\{\frac{eV_{a2}}{KT}\right\}-1\right]}{\left[\exp\left[\frac{eV_{a1}}{KT}\right]-1\right]} \right\}$$

$$\Rightarrow E_{g2}-E_{g1} = (0.0259)\ln\left\{ \frac{10\times10^{-3}}{10\times10^{-6}} \cdot \frac{\exp\left\{\frac{0.32}{0.0259}\right\}-1}{\exp\left\{\frac{0.255}{0.0259}\right\}-1} \right\}$$

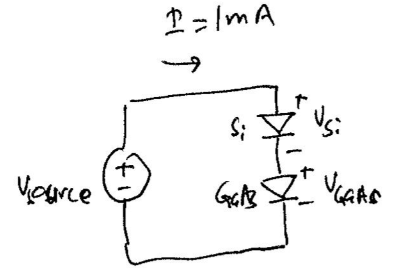
$$\Rightarrow E_{g2}-E_{g1} = 0.2349\ \text{eV}$$

$$\because E_{g1} = 0.525\text{eV}$$

$$\Rightarrow \boxed{E_{g2} = 0.769\,\text{eV}}$$ Ans.

Problem 3b

(i)



$V_{source} = V_{Si} + V_{GaAs}$

Area of both diodes: $A = 100 \times 100 \mu m^2 = 10^{-4} cm^2$

$N_A = 5 \times 10^{15} /cm^3$

$N_D = 1 \times 10^{18} /cm^3$

Circuit current $= 1mA$

~~for Vf~~ from schokley equation:

$I = I_s . \{ \exp(\frac{eV_a}{kT}) - 1 \}$

$\Rightarrow I = A.n_i^2.e \left[\frac{D_p}{L_p N_D} + \frac{D_n}{L_n N_A} \right] . \left[\exp\{ \frac{eV_a}{kT} \} - 1 \right]$

$\Rightarrow I = A.n_i^2.e. \left[\frac{D_p}{\sqrt{D_p \tau_p} N_D} + \frac{D_n}{\sqrt{D_n \tau_n} N_A} \right] . \left[\exp\{ \frac{eV_a}{kT} \} - 1 \right].$

For Silicon Diode:

$1 \times 10^{-3} = 10^{-4}.(10^{10})^2.(1.6 \times 10^{-19}) \left[\frac{12}{\sqrt{12 \times 10^{-6}}.10^{18}} + \frac{35}{\sqrt{35 \times 10^{-6}}.5 \times 10^{15}} \right] . \left\{ \exp\{ \frac{eV_a}{kT} \} - 1 \right\}$

$1 \times 10^{-3} = 1.9 \times 10^{-15} \left[\exp\{ \frac{eV_a}{kT} \} - 1 \right]$

$\Rightarrow V_a = \frac{kT}{e} . \ln \left[\frac{10^{-3}}{1.9 \times 10^{-15}} + 1 \right]$

$\Rightarrow \underline{\underline{V_a = 0.70V}} \; \{ = V_{Si} \}$

for GaAs Diode:

$1 \times 10^{-3} = 10^{-4} (2 \times 10^{6})^2 (1.6 \times 10^{-19}) \left[\frac{10}{\sqrt{10 \times 10^{-8}}.10^{18}} + \frac{220}{\sqrt{220 \times 10^{-8}}.5 \times 10^{15}} \right] . \left[\exp\{ \frac{eV_a}{kT} \} - 1 \right]$

$\Rightarrow 1 \times 10^{-3} = 1.9 \times 10^{-21} \left[\exp\{ \frac{eV_a}{kT} \} - 1 \right]$

$\Rightarrow V_a = \underline{1.06V} \; [= V_{GaAs}]$

Hence: $V_{source} = V_{Si} + V_{GaAs}$

$\Rightarrow V_{source} = \underline{\underline{1.76V}}$

(ii) The difference in forward voltages originate from the fact that the threshold voltage (voltage at which the diode turns on) is approximately equal to $\left(\frac{E_g}{e}\right)$. Since Silicon has a smaller bandgap $[E_{g_{Si}} = 1.12eV]$ compared to GaAs $[E_{g_{GaAs}} = 1.42eV]$ $\Rightarrow$ Silicon has a small turn on voltage.

(iii) $\because$ Bandgap of Ge $E_{g_{Ge}} = 0.66\,eV < E_{g(Si)} < E_{g(GaAs)}$

Hence V_a for Ge will be small than both Si and GaAs.

Problem 4a

(i)

MOSFET-1: n-channel or NMOS

MOSFET-2: p-channel or PMOS

(ii)

SUBSTRATE: p-type

WELL: n-type

(iii)

MOSFET 1:

Metal workfunction (Aluminum) $e\phi_m = 4.2\,eV$

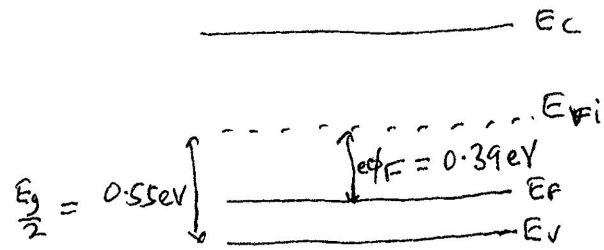
Electron Affinity of Silicon $\chi_s = 4.0\,eV$

Substrate = p-type, doping $N_A = 5\times10^{16}/cm^3$

$$\phi_F = \frac{KT}{e}.\ln\left(\frac{N_A}{n_i}\right) = 0.0259\ln\left(\frac{5\times10^{16}}{1.5\times10^{10}}\right) = 0.39\,V$$

$[\phi_F > 0$ for p-type$]$

or $E_{Fi} - E_F = 0.39\,eV$



Hence: Semiconductor work function:

$$\phi_s = \frac{\chi_s}{e} + \frac{E_g}{2e} + \phi_F = 4.0 + 0.55 + 0.39$$

$$\Rightarrow \phi_s = 4.94\,V$$

or work function difference

$$\phi_{ms} = \phi_m - \phi_s = 4.20 - 4.94 = \underline{\underline{-0.74\,V}}$$

Flat band voltage: $V_{FB} = \phi_{ms} = \underline{\underline{-0.74\,V}}$

(iv) MOSFET 2:

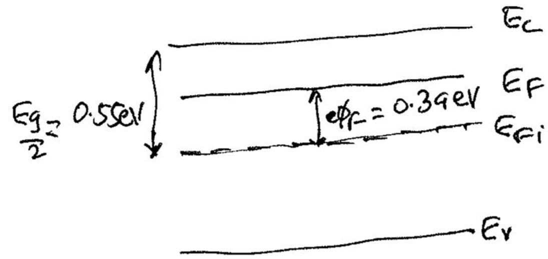
ϕ_m (Aluminium) $= 4.2\,eV$

substrate well = ~~n~~-type

$\Rightarrow$ Doping $N_D = 5\times10^{16}\,/cm^3$

$\phi_F = -\frac{kT}{e}\ln\left(\frac{N_D}{n_i}\right) = -0.39\,V$ [$\phi_F < 0$ for n-type]

or $E_F - E_{Fi} = 0.39\,eV$



$\therefore$ semiconductor work function:

$\phi_s = \frac{\chi_s}{e} + \frac{E_g}{2e} + \phi_F$

$\Rightarrow \phi_s = 4 + 0.55 - 0.39$

$\Rightarrow \phi_s = 4.16\,V$

work function difference,

$\phi_{ms} = \phi_m - \phi_s = 4.2 - 4.16 = \underline{0.04\,V}$

or flat Band voltage

$V_{FB} = \underline{0.04\,V}$

(v) MOSFET 1: Threshold voltage:

~~$V_{T,ideal} = 2\phi_F + \frac{\epsilon_{si}}{\epsilon}$~~

$V_{T,ideal} = 2\phi_F + \frac{\epsilon_{si}}{\epsilon_{ox}} \cdot t_{ox} \cdot \sqrt{\frac{2q \cdot N_A \cdot 2\phi_F}{\epsilon_s}}$

$\Rightarrow V_{T,ideal} = 2(0.4) + \frac{11.7}{3.9}\,500\times10^{-8}\sqrt{\frac{2(1.6\times10^{-19})(5\times10^{16}).2(0.4)}{11.7\times(8.85\times10^{-14})}}$

$\Rightarrow V_{T,ideal} = 2.46\,V$

$V_{T,real} = V_{T,ideal} - |V_{FB}|$

$\Rightarrow V_{T,real} = 2.46 - 0.74 = \underline{\underline{1.72V}}$

MOSFET 2: Threshold voltage:

$$V_{T,ideal} = \underbrace{-(2\phi_F)}_{\Phi_s} - \underbrace{\frac{\epsilon_{si}}{\epsilon_{ox}} \cdot t_{ox} \cdot \sqrt{\frac{2 \cdot qN_D \cdot 2\phi_F}{\epsilon_s}}}_{V_{ox}}$$

[NOTE: Both Φ_s and V_{ox} are negative]

$\Rightarrow V_{T,ideal} = -2.46V$

$\therefore V_{T,real} = -2.46 + 0.04 = \underline{\underline{-2.42}}\ V$

NOTE: MOSFET 1 → NMOS → $V_{T,real}$ is positive

MOSFET 2 → PMOS → $V_{T,real}$ is negative

} correct!

Problem 4b

Given: $A = 7\times10^{-4}\,cm^2$, $T = 300k$, $I_{total} = 0.8\,mA$, $J_{total} = I_{total}/A$

(i)

Schottky:

$$J_{total} = J_s\left\{\exp\left(\frac{eV_a}{kT}\right)-1\right\}$$

$$J_s = 4\times10^{-8}\,\frac{A}{cm^2}$$

$$\therefore V_a = \frac{kT}{e}\left[\ln\left(\frac{J_{total}}{J_s}\right)+1\right]$$

$$\therefore V_a = 0.0259\left[\ln\left[\frac{0.8\times10^{-3}}{7\times10^{-4}\times4\times10^{-8}}\right]+1\right]$$

$$\Rightarrow V_a = \underline{0.4447\text{ Volts.}}$$

P-n junction

$$J_{total} = J_s\cdot\left\{\exp\left(\frac{eV_a}{kT}\right)-1\right\}$$

$$J_s = 3\times10^{-12}\,\frac{A}{cm^2}$$

$$V_a = \frac{kT}{e}\left\{\ln\left(\frac{J_{total}}{J_s}\right)+1\right\}$$

$$V_a = 0.0259\left\{\ln\left[\frac{0.8\times10^{-3}}{7\times10^{-4}\times3\times10^{-12}}\right]+1\right\}$$

$$V_a = \underline{0.6907\text{ Volts}}$$

(ii) P-n diode: I_{total} at $T=400k$?

$$\because J_s \propto n_i^2 \propto T^3\exp\left\{\frac{-E_g}{kT}\right\} \Rightarrow \frac{J_s(400k)}{J_s(300k)} = \left(\frac{400}{300}\right)^3\cdot\frac{\exp\left\{\frac{-1.12}{0.0259\times\frac{400}{300}}\right\}}{\exp\left\{\frac{-1.12}{0.0259}\right\}}$$

$$\Rightarrow J_s(400k) = 3.52\times10^{-7}\,\frac{A}{cm^2}$$

$$\therefore I_{total}(400k) = A\cdot J_s(400k)\cdot\exp\left\{\frac{eV_a}{kT_{400k}}\right\} = 7\times10^{-4}\times3.52\times10^{-7}\times\exp\left\{\frac{0.6907}{0.0259\times\frac{400}{300}}\right\}$$

$$\Rightarrow I_{total}(400k) = \underline{119\,mA}$$

Schottky: I_{total} at $T=400k$?

$$J_s \propto T^2\cdot\exp\left\{\frac{-e\phi_B}{kT}\right\} \Rightarrow \frac{J_s(400k)}{J_s(300k)} = \left(\frac{400}{300}\right)^2\cdot\frac{\exp\left\{\frac{-0.82}{0.0259\times\frac{400}{300}}\right\}}{\exp\left\{\frac{-0.82}{0.0259}\right\}}$$

$$\Rightarrow J_s(400k) = 1.95\times10^{-4}\,\frac{A}{cm^2}$$

$$\therefore I_{total}(400k) = A\cdot J_s(400k)\cdot\exp\left\{\frac{eV_a}{kT_{400k}}\right\} = 7\times10^{-4}\times1.95\times10^{-4}\times\exp\left\{\frac{0.4447}{0.0259\left(\frac{400}{300}\right)}\right\}$$

$$\Rightarrow I_{total} = \underline{53.42\,mA}$$