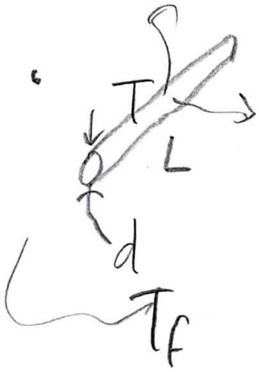
IIA 3A6 : Crib 2022 | Heat and Mass Transfer

1. 

(a) convection balances radiation

$$h\underbrace{\pi d L}(T-T_f) = \varepsilon\sigma T^4 \underbrace{\pi d L}$$

$$\Delta T = T - T_f = \frac{\varepsilon\sigma T^4}{h}$$

$$Nu = \frac{hd}{\lambda} = 0.35\left(\frac{Ud}{\nu}\right)^{0.5}$$

$$\Delta T = \frac{\varepsilon\sigma T^4}{0.35\left(\frac{Ud}{\nu}\right)^{0.5}\frac{\lambda}{d}}$$

~~16×10^{12}~~
~~911,840~~
2,605,257 ✓
$\frac{1}{10^{+7}}$

$$\therefore \Delta T = \frac{1\times 5.699\times10^{-8}\ 2000^4}{0.35\left(\frac{10\times10\times10^{-6}}{10\times10^{-5}}\right)^{0.5}\frac{100}{10\times10^{-6}}} = 0.26\ K$$

(b)  unsteady $h\pi d L(T_f - T) = \rho c \frac{\pi d^2 L}{4}\cdot\frac{dT}{dt}$

$$\therefore \underline{\frac{dT}{T-T_f} = -\frac{dt}{\tau}} \text{ where } \underline{\tau = \rho c d/4h}.$$

Max. frequency, $f \sim \frac{1}{\tau}$

$$Nu = 0.35\,Re_d^{0.5} = 0.35\left(\frac{10\times10\times10^{-6}}{10\times10^{-5}}\right)^{0.5} = 0.35 = hd/\lambda_f$$

$$h = 0.35\left(\frac{100}{10^{-5}}\right) = 0.35\times10^{7}\ W/m^2K$$

$$\therefore \underline{f} = \frac{4\times0.35\times10^{7}}{4500\times500\times10.10^{-6}} = \underline{0.62\ MHz}.$$

(c) for two wavelengths at a given temperature

$$\frac{I_{b2}}{I_{b1}} = \left(\frac{\lambda_1}{\lambda_2}\right)^5 \cdot \frac{\exp(C_2/\lambda_1 T)}{\exp(C_2/\lambda_2 T)} = \left(\frac{\lambda_1}{\lambda_2}\right)^5 \exp\left[\frac{C_2}{T}\left(\frac{1}{\lambda_1}-\frac{1}{\lambda_2}\right)\right]$$

$$\therefore\ T = \frac{c\left(\frac{1}{\lambda_1}-\frac{1}{\lambda_2}\right)}{\log\Big[\underbrace{\frac{I_{b2}}{I_{b1}}}_{R_I}\underbrace{\left(\frac{\lambda_2}{\lambda_1}\right)^5}_{R_\lambda}\Big]}$$

T = fn (2x intensities at 2x wavelengths).

(d)

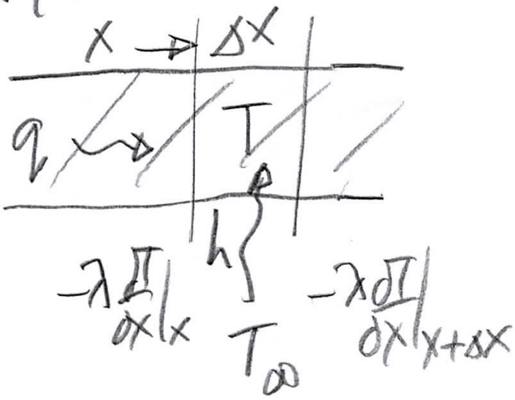
$$\frac{\partial T}{\partial R_I} = c\left(\frac{1}{\lambda_1}-\frac{1}{\lambda_2}\right)\frac{-1}{(\log[..])^2}\cdot\frac{\cancel{R_\lambda^5}}{R_I \cancel{R_\lambda^5}}$$

$$\therefore\ \frac{1}{T}\frac{\partial T}{\partial R_I} = -\frac{1}{\log(R_I \cdot R_\lambda^5)}\cdot\frac{1}{R_I}$$

$$\therefore\ \frac{dT}{T} = -\frac{1}{\log(..)}\cdot\frac{dR_I}{R_I}$$

So, choose R_λ as large as possible to minimize inaccuracy.

2. (a)



$$A\left[\left(-\lambda\frac{dT}{dx}\right)_x - \left\{(\ldots)_x + \frac{d}{dx}(\ldots)_x \Delta x\right]\right.$$
$$-hP\Delta x(T-T_\infty) = 0$$

$$\therefore \lambda A \frac{d^2T}{dx^2}\Delta x - hP\Delta x(T-T_\infty) = 0 \rightarrow \frac{d^2\theta}{dx^2} - \underbrace{\frac{hP}{\lambda A}}_{m^2}\theta = 0 \text{ with } \theta = T-T_\infty$$

$$\theta = A\cosh(mx) + B\sinh(mx)$$

$\theta(0) = \theta_0 \rightarrow A = \theta_0$

$\theta(L) = 0 \rightarrow B = -A/\tanh(mL)$

$$\therefore \quad \underline{T - T_\infty = (T_0 - T_\infty)\left[\cosh(mx) - \frac{\sinh(mx)}{\tanh(mL)}\right]}$$

(b) 3D conduction $\nabla^2 T = 0 = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}$

for $x^2 \sim L^2 \gg A$ and $y, z \sim \sqrt{A} \ll L$ from (a)

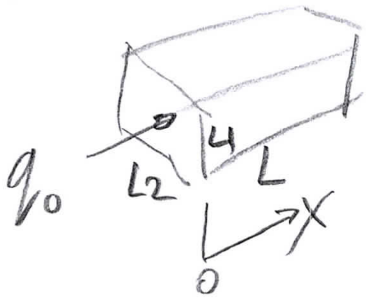
$\frac{\partial^2 T}{\partial x^2} \ll \frac{\partial^2 T}{\partial y^2}, \frac{\partial^2 T}{\partial z^2}$ so $\underline{\frac{\partial^2 T}{\partial x^2} \sim 0}$

$\therefore$ with convective heat transfer $\frac{d^2T}{dx^2} = \frac{hP}{\lambda A}(T - T_\infty)$

so $\frac{\cancel{\Delta T}}{L^2} \sim \frac{hP\cancel{\Delta T}}{\lambda A} \Rightarrow \frac{hP}{\lambda}\cdot\frac{L^2}{A} \sim 1$

Hence, as $\frac{L^2}{A} \gg 1$ then $\underline{\frac{hP}{\lambda} \ll 1}$.

(c) fin volume $V = L_1 L_2 L$



$$q_0 = -\lambda \frac{dT}{dx}\Big|_{x=0} = \frac{-\lambda \theta_0 \overset{\sqrt{\frac{hP}{\lambda A}}}{m}}{\tanh mL}$$

$\left.\begin{matrix}\sinh 0 = 0 \\ \cosh 0 = 1\end{matrix}\right\}$

perimeter $P = 2(L_1 + L_2) = 2\left[L_1 + \frac{V}{L}\cdot\frac{1}{L_1}\right]$

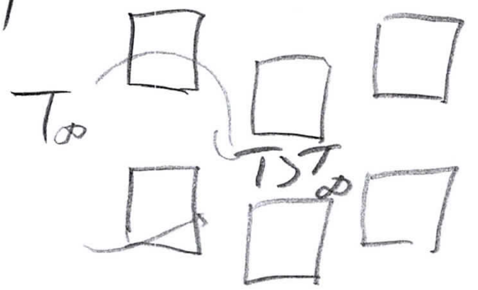
$$A = L_1 L_2 = V/L$$

$$m = \sqrt{\frac{hP}{\lambda A}} = \sqrt{\frac{h}{\lambda}}\sqrt{\frac{2L}{V}\left(L_1 + \frac{V}{L}\cdot\frac{1}{L_1}\right)} \quad \text{so} \quad Q_0 = \overset{A}{\frac{V}{L}}\;\overbrace{\frac{\lambda\theta_0}{\tanh mL}\cdot\sqrt{\frac{2Lh}{\lambda V}}\sqrt{L_1 + \frac{V}{L}\cdot\frac{1}{L_1}}}^{q}$$

^{2} easier...

$$\frac{dQ_0^2}{dL_1} = 0 = \text{const.}\left(1 + \frac{V}{L}\left(-\frac{1}{L_1^2}\right)\right) = 0, \quad L_1^2 = V/L \to \underline{L_1 = L_2 = \sqrt{\frac{V}{L}}}.$$

(d)

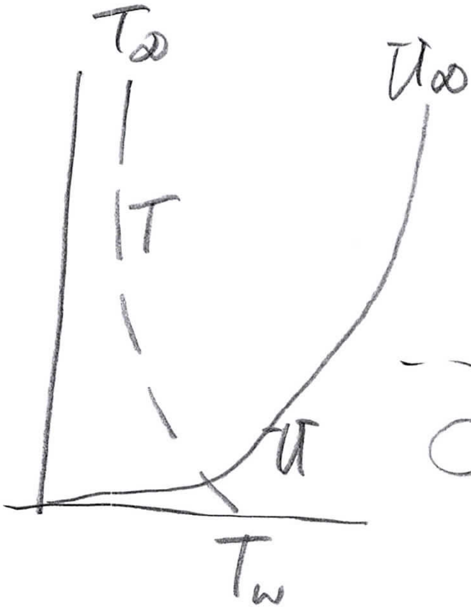


arrays of square fins can be used but their effectiveness depends on cross-fin effects — downstream fins less effective in wakes of upstream ones. Other factors include ease of manufacture

(e)

similar (d) but especially interference effects from wakes of upstream interacting with those downstream.

3.



(a) turbulent layer

$u(t) = U + u'$

$v(t) = V + v'$

$T(t) = T + T'$

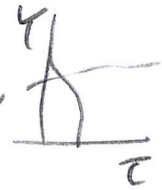
$$\begin{cases} u' \sim l \dfrac{dU}{dY} & v' \sim u' \\ T' \sim l \dfrac{dT}{dY} & \text{mixing length } l \end{cases}$$

(b)

$$\tau \sim \overline{m'u'} \quad ; \quad m' \sim \rho v'$$

$$\therefore \tau/\rho \sim -\overline{u'v'} = l^2 \left(\frac{dU}{dY}\right)^2$$

Prandtl: $l = kY$ and for equilibrium layer $\tau \sim \tau_w$



$$\therefore \sqrt{\tau_w/\rho} = kY \frac{dU}{dY}$$

$$\therefore \frac{U}{\sqrt{\tau_w/\rho}} = \frac{1}{k} \log Y + C$$

$$\sqrt{\frac{\frac{1}{2}\rho U_\infty^2 C_f}{\rho}} = U_\infty \sqrt{\frac{C_f}{2}}$$

$$\therefore \frac{U}{U_\infty} = \frac{1}{k\sqrt{\frac{C_f}{2}}} \log Y + C$$

(c) similarly $\quad q \sim \overline{m' c_p T'}$

$$\therefore q/\rho c_p \sim \overline{v'T'}$$

$$\therefore q/\rho c_p = \ell^2 \frac{dU}{dY}\cdot\frac{dT}{dY}$$

$\frac{dU}{dY}$ from (b)

setting $q \sim q_w$ $\quad \frac{q_w}{\rho c_p} = \kappa^2 Y^2 \cdot \frac{U_\infty}{\kappa}\sqrt{\frac{2}{c_f}}\,\frac{1}{Y}\cdot\frac{dT}{dY}$

$$\therefore \frac{q_w}{\rho c_p U_\infty}\cdot\sqrt{\frac{c_f}{2}} = \kappa Y \frac{d}{dY}(T - T_w)$$

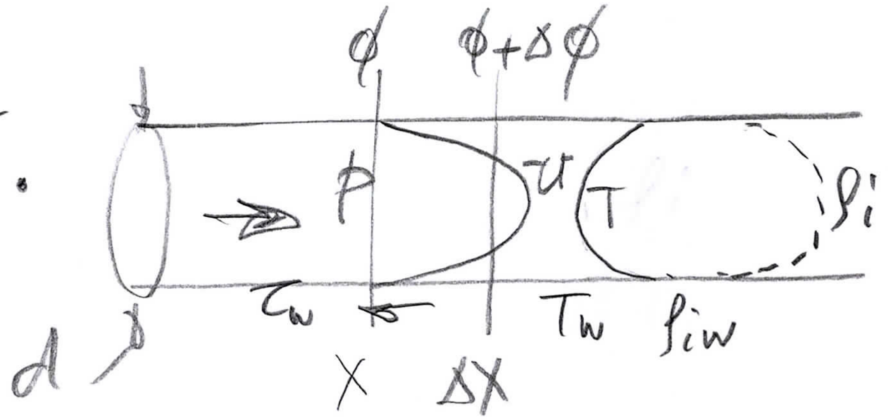
$q_w = h(T_\infty - T_w)$ $\therefore \frac{h}{\rho c_p U_\infty}\sqrt{\frac{c_f}{2}} = \kappa Y \frac{d}{dY}\left(\frac{T-T_w}{T_\infty - T_w}\right)$

Stanton number St

$$\therefore \frac{T - T_w}{T_\infty - T_w} = \frac{St}{\kappa}\sqrt{\frac{c_f}{2}}\log Y + D$$

As expected, a "similar" solution derived from the similarity in the basic equations of motion.

4.



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(a) $\frac{\pi d^2}{4} \Delta p + \tau_w \pi d \Delta X = 0 \rightarrow \underline{\frac{dp}{dX} = - \tau_w \frac{4}{d}}$.

(b) $\rho u_b \frac{\pi d^2}{4} c_p \Delta T_b = q_w \pi d \Delta X$

$h = \frac{q_w}{T_b - T_w}$ — Q/A

Stanton, St

$\rightarrow \underline{\frac{dT_b}{dX} = - \frac{h}{\rho c_p u_b} \cdot (T_b - T_w) \cdot \frac{4}{d}}$

(c) $u_b \frac{\pi d^2}{4} \Delta \rho_{ib} + g_{di} \pi d \Delta X = 0$

$h_{mi} = \frac{g_{di}}{\rho_{iw} - \rho_{ib}}$ — $\dot{m}_i/A$

$\rightarrow \underline{\frac{d\rho_{bi}}{dX} = - \frac{h_{mi}}{u_b} \cdot (\rho_{ib} - \rho_{iw}) \cdot \frac{4}{d}}$.

(d) At the wall, $h_{mi}(\rho_{ib}-\rho_{iw}) = \eta \dot{R}_i$

η = surface density & texture factor

$\dot{R}_i$ = reaction rate

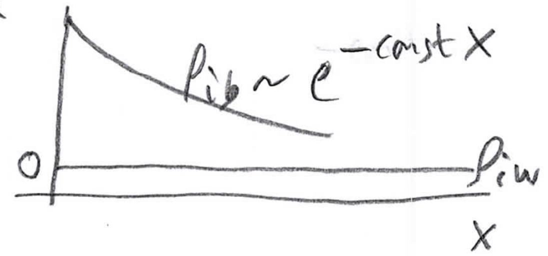
(typically $\dot{R}_i = c\,\rho_{is}\,\rho_{O_2 s}$, $c = fn(T)$).

Two limit cases:

(i) Diffusion limited surface — fully warmed up

when temperature is highest, so is the reaction so the species concentration becomes independent of the reaction rate → Sh, Re, Pr, Sc ... → h_m = constant

$$\rightarrow \frac{d\rho_{ib}}{dx} + \text{const.}(\rho_{ib}-\rho_{iw}) = 0$$



(ii) Constant surface reaction rate — device running cold

$$h_{mi}(\rho_{iw}-\rho_{ib}) = \eta \dot{R}_i = \text{constant} \rightarrow \frac{d\rho_{ib}}{dx} + \text{constant} = 0$$