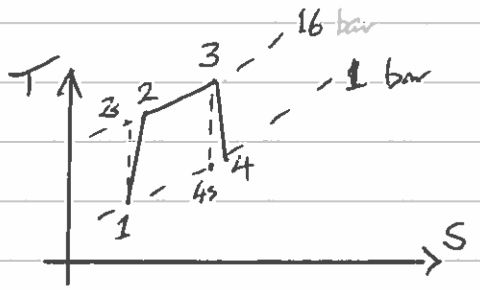
a)



(i) $T_1 = 288\,K$, $T_{2s} = (P_2/P_1)^{\frac{\gamma-1}{\gamma}} T_1 = 635.96\,K$

Compressor work input $= \dfrac{h_{2s}-h_1}{\eta_c} = \dfrac{c_p(T_{2s}-T_1)}{\eta_c} = \underline{\underline{437(.12)\ kJ/kg}}$

(ii) $T_3 = 1400\,K$, $T_{4s} = (P_1/P_2)^{\frac{\gamma-1}{\gamma}} T_3 = 634.01\,K$

Turbine work output $= \eta_t (h_3 - h_{4s}) = 692.84\ kJ/kg$

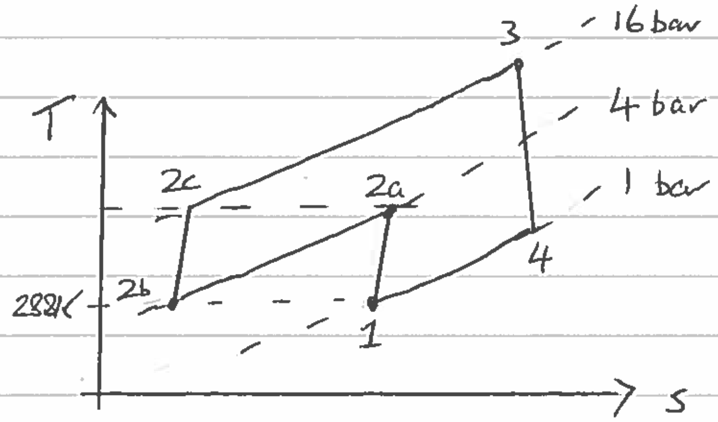
Net work output $= 692.84 - 437.12 = 255.72 \Rightarrow \underline{\underline{256\ kJ/kg}}$

(iii) $c_p T_2 = c_p T_1 + 437.12 = 726.56\ kJ/kg$

Heat supplied $= c_p(T_3 - T_2) = 680.44\ kJ/kg$

$\therefore \quad \eta_{th} = 255.72/680.44 = \underline{\underline{0.376}}$

b)



(i) $T_{2as} = 4^{\gamma-1/\gamma} T_1 = 427.97\,K$; likewise T_{2cs} $(T_{2b} = T_1)$

$h_{2a} - h_1 = \dfrac{c_p(T_{2as}-T_1)}{\eta_c} = 175.83\ kJ/kg$; likewise $h_{2c} - h_{2b}$

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Question number 1 Sheet number 2

Hence total compression work $= \underline{352 \text{ kJ/kg}}$

(ii) New heat input $= h_3 - h_{2c} = c_p(T_3 - \underbrace{T_{2b}}_{T_1}) - \underbrace{c_p(T_{2c} - T_{2b})}_{h_{2c} - h_{2b}}$

$= 941.73$ i.e. $\underline{942 \text{ kJ/kg}}$

$\therefore \eta_{th} = (692.84 - 351.66)/941.73 = \underline{\underline{0.362}}$

(iii) No benefit (in fact, slightly worse) for efficiency, but more work output for same size turbine. Assessment changes if source from waste heat, because then additional heat input would otherwise be thrown away.

c) Isothermal compression: $q - w_x = 0$ SFEE

Reversible steady-flow heat extraction: $q = T\Delta s$

2nd law for control volume $= -RT\ln(P_2/P_1)$

$= -229.17 \text{ kJ/kg}$

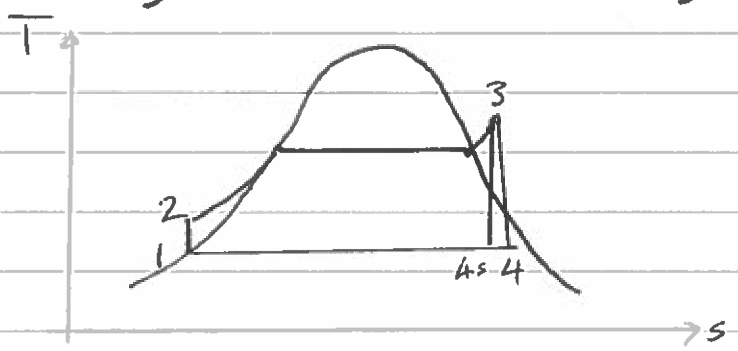
Compressor work input is $\dfrac{229.17}{0.8} = 286.46 \text{ kJ/kg}$

Turbine output unchanged at 692.84 kJ/kg

$\Rightarrow$ Net work output $\underline{\underline{406 \text{ kJ/kg}}}$

a)(i) Boiler exit: 200°C, 5 bar is single-phase steam

Condenser entry: will be two-phase, by inspection of h-s chart



(ii) NB Tabulated values used here for reference.
Chart accuracy acceptable for exam answers.

$h_1 = 191.8$ kJ/kg $\qquad h_3 = 2855.8$ kJ/kg

$s_1 = 0.649$ kJ/kgK $\qquad s_3 = 7.0610$ kJ/kgK

$v_1 = 0.001010$ m³/kg

Pump work (ideal): $-w_x = h_2 - h_1 = \int_1^2 v\,dp \approx v_1 \Delta p = 0.5$ kJ/kg

$\Rightarrow h_2 = 192.3$ kJ/kg.

Heat input: $q = h_3 - h_2 = 2663.5$ kJ/kg

Turbine work: $x_{4s} = \dfrac{7.061 - 0.649}{8.149 - 0.649} = 0.8549 \Rightarrow h_{4s} = 2236.9$ kJ/kg

$\therefore$ Turbine work $= -0.85 \times (2236.9 - 2855.8) = 526.1$ kJ/kg

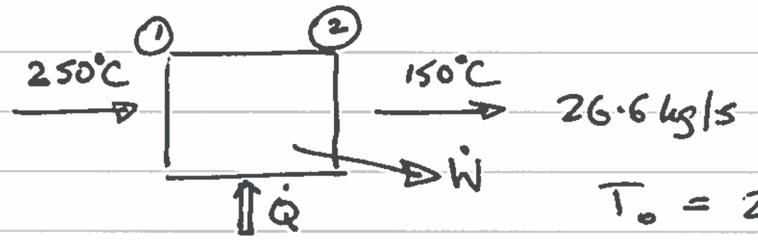
and $h_4 = 2855.8 - 526.1 = 2329.7$ kJ/kg.

$\dot{m} = 1$ kg/s

Heat input $\underline{\underline{2664 \text{ kW}}}$ $\qquad$ Power output $(526.1 - 0.5) \times 1 = \underline{\underline{526 \text{ kW}}}$

b) (i) $\dot{m}_{gas} c_p \Delta T = 2663.5 \text{ kW} \Rightarrow \dot{m}_{gas} = \underline{\underline{26.6(35) \text{ kg/s}}}$

(ii)



2nd law for CV: (max work ⇒ reversible)

$$\dot{m}\left[c_p \ln \frac{T_2}{T_1}\right] = \frac{\dot{Q}}{T_0}$$

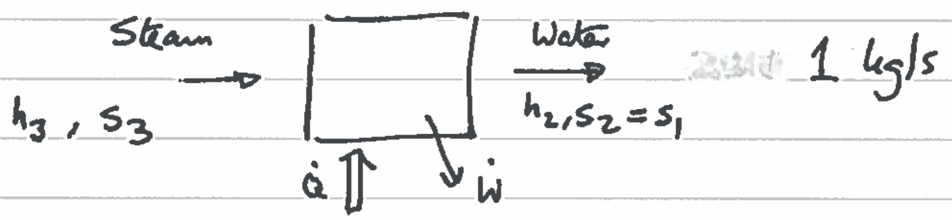
[1,2 refer to current diag.]

* Working directly from availability also OK

$$\Rightarrow \dot{Q} = -1656.4 \text{ kW}$$

1st law: $\dot{W} = \dot{Q} - \dot{m}(h_2 - h_1) = \underline{\underline{1007 \text{ kW}}}$

c) (i)



[2,3 as T-s diag.]

2nd law: $\dfrac{\dot{Q}}{T_0} = 1 \times (s_2 - s_3) \Rightarrow \dot{Q} = -1879.7 \text{ kW}$

1st law: $\dot{W} = \dot{Q} - \dot{m}(h_2 - h_3) = \underline{\underline{784 \text{ kW}}}$

(ii) 784 kW less than b)(ii) because of irreversibility in heat transfer from waste gases in boiler.

Actual output in a)(ii) < 784 kW because of irreversibility in turbine and in condenser heat transfer.

* N.B. OK to use 'availability' except in (b) and (c).

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Question number 3 Sheet number 1

a) (i) U : effective heat-transfer coefficient

A : area over which heat transfer occurs

(ii) $$\Delta T_m = \frac{\Delta T_1 - \Delta T_2}{\ln\left(\frac{\Delta T_1}{\Delta T_2}\right)}$$ where 1, 2 are end stations, and ΔTs the temperature differences between the streams

$\Delta T_1 = 60-40 = 20°C$ $\Delta T_2 = 55-15 = 40°C$

$\Rightarrow$ $\Delta T_m = \underline{\underline{28.9°C}}$; $UA = \underline{\underline{69.3 \text{ W/°C}}}$

b) $\dot{Q} = \dot{m} c_p \Delta T$ for each stream $\Rightarrow$ $\dot{m}_{air} = \underline{\underline{0.0796 \text{ kg/s}}}$

$\dot{m}_{water} = \underline{\underline{0.0956 \text{ kg/s}}}$

c) Irrespective of how big UA is....
- max possible air outlet temperature 60°C (A)
- min possible water outlet temperature 15°C (B)

Case (A) corresponds to $\dot{Q}$ = 3.6kW

Case (B) — " — = 18kW

Lower applies (case (B) breaches air outlet limit) $\Rightarrow$ Air outlet $\underline{\underline{60°C}}$

Water outlet (via $\dot{m} c_p \Delta T$) : $\underline{\underline{51°C}}$

N.B. Also OK to argue from slopes on T–Q plot.

d) (i) New $\dot{Q} = \dot{m}_{water} c_{p\,water} \Delta T_{water} = 2.4\text{kW}$

$\therefore$ Expect $\Delta T_m = \dot{Q}/UA = 34.62°C$ (*)

Check : new $\Delta T_1 = 60 - 29.4 = 30.6°C$ $\Delta T_2 = 54 - 15 = 39°C$

giving $\Delta T_m = 34.63°C$ in agreement with (*)

$$\dot{m}_{air} = \frac{2.4 \times 10^3}{1.005 \times 10^3 \times (29.4 - 15)} = \underline{0.166 \text{ kg/s}}$$

(ii) As $\dot{m}_{air} \to \infty$, air exit temperature approaches 15°C.

In this limit, we have water stream passing through heat exchanger in finite time and exchanging heat with 15°C environment.

We have a finite rate of heat transfer, and hence a bound on water exit temp. Lower bound, because the heat transfer is the maximum attainable with this approach. (15°C is minimum possible air temp.)

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Question number 4 Sheet number 1

a) Flow steady and inviscid $\Rightarrow$ Bernoulli applies.

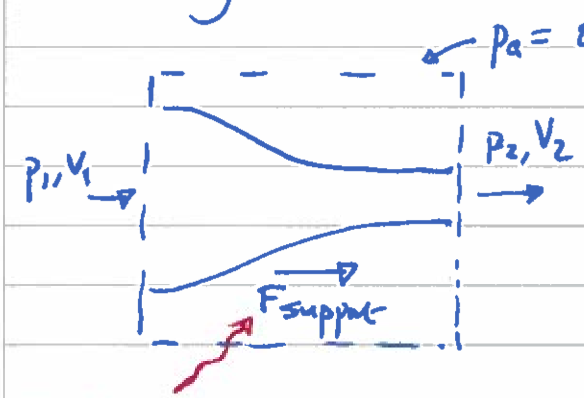
Entry: $p_1 = p_s - \frac{1}{2}\rho\left(\frac{Q}{A_1}\right)^2$ $\quad [V_1 = Q/A_1]$

Exit: $p_2 = p_s - \frac{1}{2}\rho\left(\frac{Q}{A_2}\right)^2$ $\quad [V_2 = Q/A_2]$

b)(i)

Apply momentum to control volume around contraction:

Inside contraction also possible; then requires additional force-balance argument



bracket force on contraction

$$F_{support} + p_1A_1 - p_2A_2$$

$$= \rho Q\left[V_2 - V_1\right]$$

$$= \rho Q^2\left[\frac{1}{A_2} - \frac{1}{A_1}\right]$$

Apply pressure formulae from (a):

$$F_{support} + p_s(A_1 - A_2) - \frac{1}{2}\rho\left[\frac{Q^2}{A_1} - \frac{Q^2}{A_2}\right] = \rho Q^2\left[\frac{1}{A_2} - \frac{1}{A_1}\right]$$

$$F_{support} = -p_s(A_1 - A_2) + \frac{1}{2}\rho Q^2\,\frac{A_1 - A_2}{A_1A_2}$$

$$= (A_1 - A_2)\left[-p_s + \frac{\rho Q^2}{2A_1A_2}\right]$$

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Question number 4 Sheet number 2

b)(ii) Zero support force when $p_s = \dfrac{\rho Q^2}{2A_1A_2}$

(iii) $p_s > \dfrac{\rho Q^2}{2A_1A_2}$ implies $F_{support} < 0$, ~~ie~~

i.e. in direction opposing flow.

c)(i) Now we have $p_2 = \left(p_s - k.\frac{1}{2}\rho \frac{Q^2}{A_1^2}\right) - \frac{1}{2}\rho \frac{Q^2}{A_2^2}$

$$= p_2^* - \frac{k}{2}\rho \frac{Q^2}{A_1^2} \qquad p_2^* = p_s - \frac{1}{2}\rho\frac{Q^2}{A_2^2}$$

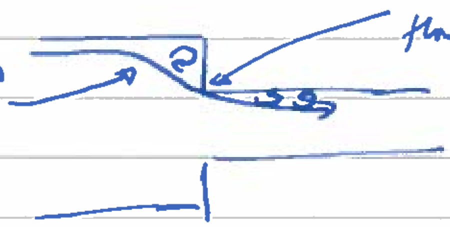
Momentum again: $F_{support} + p_1A_1 - p_2^*A_2 + \frac{k}{2}\rho \frac{Q^2 A_2}{A_1^2} = \rho Q(V_2 - V_1)$.

For p_s as b(ii), we have $p_1A_1 - p_2^*A_2 = \rho Q(V_2 - V_1)$

(since support force was then zero).

$\therefore$ $F_{support} = -\frac{1}{2}\rho Q^2 . \frac{kA_2}{A_1^2}$, $\therefore$ opposing flow.

(ii) Loss is associated with viscous effects, which arise here because there are regions of flow separation with high shear/mixing:

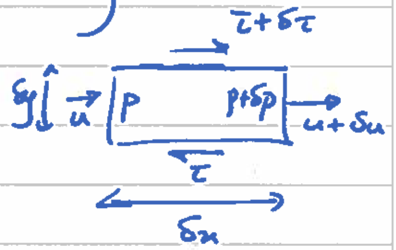


Increasing p as flow approaches corner stagnation causes separation.

flow unable to traverse sharp corner $\Rightarrow$ separates.

a) • Parallel streamlines $\Rightarrow$ p is fn(x) only

Consideration of elemental control volume:



• Mass conservation gives $\delta u = 0$

• Momentum (steady) gives $-\delta p\,\delta y + \delta\tau\,\delta x = \rho\,\delta y\,(\cancel{\delta u})^{0}$

leading to $\dfrac{\partial\tau}{\partial y} = \dfrac{dp}{dx}$

• Newton's law of viscosity: $\tau = \mu\dfrac{du}{dy}$, fn(y) only

$$\Rightarrow \quad \mu\frac{d^2u}{dy^2} = \frac{dp}{dx} = \text{const.}$$

• Integrate twice with no-slip $(u=0)$ on $y=\pm h$.

b) Continuity: $2HU_p = Q$, where Q is gap flow rate

$$Q = \int_{-h}^{h} u(y)\,dy = \frac{h^3}{2\mu}\left(-\frac{dp}{dx}\right)\int_{-1}^{1}(1-\eta^2)\,d\eta \qquad \eta = y/h$$

$$= \frac{h^3}{\mu}\left(-\frac{dp}{dx}\right)\int_{0}^{1}(1-\eta^2)\,d\eta$$

$$= \frac{2h^3}{3\mu}\left(-\frac{dp}{dx}\right)$$

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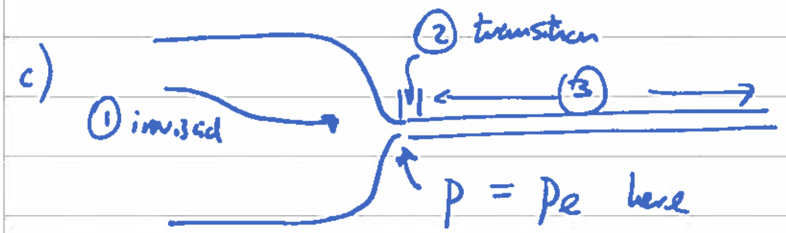
Question number 5 Sheet number 2

b) cont'd ... Hence $U_p = \frac{h^3}{3\mu H}\left(-\frac{dp}{dx}\right)$

Now $-\frac{dp}{dx} = \frac{p_r}{L}$ (since constant)

so $p_r = \frac{3\mu H L}{h^3} U_p$

c)



③ is fully-developed viscous flow.

Continuity still holds, as does relation between Q and dp/dx, but now $-dp/dx = p_e/L$.

① effectively inviscid ⇒ Bernoulli gives $p_r + \frac{1}{2}\rho U_p^2 = p_e + \frac{1}{2}\rho U_p^2\left(\frac{H}{h}\right)^2$

$\boxed{\rho : \text{fluid density}}$

Entry velocity $= U_p H/h$ from continuity

and (from (b)) $p_r = \underbrace{\frac{3\mu H L}{h^3} U_p}_{p_e} + \frac{1}{2}\rho U_p^2\left[\left(\frac{H}{h}\right)^2 - 1\right]$

Factorise : $p_r = \frac{3\mu H L}{h^3} U_p\left[1 + \frac{\rho U_p h}{6\mu H L}(H^2 - h^2)\right]$

As $H^2 \gg h^2$, we need $\frac{\rho U_p h H}{\mu L}$ small for (b) to be valid.

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Question number 6 Sheet number 1

a)

Total head change = 0 overall

Hence pump must give total pressure increase equal to losses in connecting pipe and in entry to delivery tank.

Pipe : $\Delta p_T = \Delta p = \frac{1}{2}\rho V^2 . 4c_f \frac{L}{d}$

Tank entry : $\Delta p_T = \frac{1}{2}\rho V^2$ Easily forgotten!

$V = \frac{4Q}{\pi d^2} = \frac{4 \times 0.01}{\pi (0.1)^2} = 1.2732 \text{ m/s}$; $\frac{1}{2}\rho V^2 = 810.57 \text{ Pa}$

$\Delta p_T = \left(1 + 4c_f \frac{L}{d}\right) \frac{1}{2}\rho V^2 = 13.780 \times 10^3 \text{ Pa}$

Power requirement $= Q\,\Delta p_T = \underline{\underline{138 \text{ W}}}$

b) Δp_T is unchanged (pipe 1 conditions) at 13.78×10^3 Pa

Considering flow from pump to tank 2:

$\Delta p_T = \left(1 + 4c_f \frac{L}{d} + K\right) \frac{1}{2}\rho V_2^2$, with $\frac{1}{2}\rho V_2^2 = 202.63 \text{Pa}$

$\Rightarrow \underline{\underline{K = 51}}$

c) Pipe 1 : $\Delta p_T = \left(1 + \frac{4 c_f L}{d}\right) \frac{1}{2}\rho V_1^2 = 17 \times \frac{1}{2}\rho V_1^2$

Pipe 2 : $\Delta p_T = \left(1 + \frac{4 c_f L}{d} + K\right) \frac{1}{2}\rho V_2^2 = 68 \times \frac{1}{2}\rho V_2^2$

Hence $V_2 = \frac{1}{2} V_1$; equally $Q_2 = \frac{1}{2} Q_1$

Pump power $= \Delta p_T (Q_1 + Q_2) = \Delta p_T \left(\frac{3}{2} Q_1\right)$

$$= \Delta p_T \left(\frac{3 \pi d^2 V_1}{8}\right)$$

i.e. $137.80 = 17 \times \frac{1}{2}\rho V_1^2 \times \frac{3 \pi d^2 V_1}{8}$

giving $V_1 = 1.1123$ m/s ; $\underline{\underline{Q_1 = 8.74 \times 10^{-3} \text{ m}^3/\text{s}}}$

$\underline{\underline{Q_2 = 4.37 \times 10^{-3} \text{ m}^3/\text{s}}}$