METIIA Paper 3 2024/2025 Cribs

Question 1.

(a) Annual Demand	5000
Purchase Cost	£100
Holding Cost	£30
Ordering Cost	£1

EOQ 18

Carrying Cost	£273.86
Ordering Cost	£273.86
Purchase Cost	£500,000.00
Total Cost	£500,547.72

Operating Days 312

No. of Orders 278

No. of Operating Days between Orders 1.12
Order 5 days a week (Monday-Friday) and every third Saturday

(b) EOQ is based on several key assumptions:

Constant Demand: EOQ assumes that demand for the product is constant over the time period being considered. This means that there are no fluctuations or variability in demand during the lead time.

Constant Unit Cost: The unit cost of the product remains constant regardless of the order quantity. This assumption simplifies the calculation of inventory costs and allows for a straightforward analysis of ordering and holding costs.

Instantaneous Replenishment: EOQ assumes that inventory is replenished instantly when the stock reaches zero. There is no delay or lead time between placing an order and receiving the inventory.

No Quantity Discounts: The model assumes that there are no quantity discounts offered by suppliers for purchasing larger quantities of the product. This ensures that the analysis focuses solely on minimizing total inventory costs rather than considering discounts.

No Stockouts or Shortages: EOQ assumes that there are no stockouts or shortages during the replenishment lead time. This means that the inventory is always available to meet customer demand without interruption.

No Holding or Storage Constraints: The model assumes that there are no constraints on holding or storing inventory, such as limited warehouse space or expiration dates. This allows for the calculation of holding costs based solely on the average inventory level.

Fixed Ordering and Holding Costs: EOQ assumes that ordering costs and holding costs are fixed and do not change with the order quantity. This simplifies the analysis by allowing for the calculation of total inventory costs based on a consistent set of parameters.

In practice:

- Costs are difficult to measure.
- Demand is not constant.
- Lack of focus on inventory control.
- Need to focus on reducing setup costs and holding costs to reduce EOQ and total costs.

(c) For upstream manufacturers, there are several benefits if a retail store holds safety stock:

Stabilises Demand Fluctuations: When retailers hold safety stock, they are better equipped to handle fluctuations in customer demand without immediately adjusting their orders to manufacturers. This helps to stabilize demand for upstream manufacturers, reducing the variability in order quantities and production schedules.

Reduced Bullwhip Effect: The bullwhip effect refers to the amplification of demand variability as it moves upstream in the supply chain. When retailers hold safety stock, they are less likely to place erratic or large orders in response to sudden changes in customer demand. This can help mitigate the bullwhip effect, leading to smoother and more predictable order patterns for upstream manufacturers.

Improved Production Planning: Safety stock held by retailers provides a buffer against demand variability and uncertainty. This allows upstream manufacturers to better plan their production schedules and allocate resources more efficiently. With greater visibility into future demand trends, manufacturers can optimize their production processes and minimize the risk of overproduction or underproduction.

Reduced Rush Orders: In situations where retailers experience unexpected spikes in demand, safety stock can help mitigate the need for rush orders to replenish inventory quickly. By having buffer inventory on hand, retailers can absorb short-term fluctuations in demand without placing undue pressure on upstream manufacturers to expedite production or delivery.

Enhanced Supplier Relationships: When retailers hold safety stock, they are less likely to place last-minute or urgent orders with upstream manufacturers. This can lead to more stable and predictable order patterns, fostering stronger relationships between retailers and their suppliers. Manufacturers may be more willing to collaborate with retailers on issues such as lead times, pricing, and product quality, knowing that demand fluctuations are being managed effectively.

(e) Holding safety stock leads to **higher inventory levels**, which may result in **increased energy usage for storage**, especially if the store uses **climate-controlled environments**. Additional inventory may require **more storage space**, potentially **impacting land use** if expansion is needed. It also increases the likelihood of **overstocking**, which can lead to increased emissions during manufacturing, transportation and disposal processes.

Extra inventory **ties up raw materials** (plastics, metals, etc.) in unused products, which might contradict circular economy principles by delaying their utilisation.

If demand does not materialise as forecasted, excess stock might lead to **product obsolescence, increasing waste**. If unsold printers become outdated or defective, disposal or recycling challenges may arise, adding to **e-waste concerns**.

(d) Holding safety stock **provides retailers with a safety net** to navigate uncertainties in demand, supply, and production - helping them maintain customer satisfaction with supply levels.

Retailers will hold safety stock, also known as buffer stock, to counter:

Demand Variability: Safety stock serves as a buffer against fluctuations or variability in customer demand. Demand for products can be unpredictable due to factors such as seasonality, promotions, or changes in consumer preferences. By holding safety stock, retailers can mitigate the risk of stockouts and ensure that they can meet customer demand even during periods of unexpected high demand.

Supplier Lead Time Variability: Suppliers may experience variability in lead times for delivering inventory. Delays in shipments due to transportation issues, production delays, or other factors can disrupt the replenishment process. Safety stock helps retailers bridge the gap between when inventory is depleted and when new stock arrives, reducing the risk of stockouts during longer-than-expected lead times.

Production Variability: For retailers who manufacture their own products or rely on suppliers for manufacturing, production variability can impact inventory levels. Fluctuations in production output due to equipment breakdowns, labour shortages, or other factors can lead to supply disruptions. Safety stock provides a cushion to absorb these fluctuations and maintain consistent inventory levels.

Demand Forecasting Errors: Despite retailers' best efforts, demand forecasts are not always accurate. Forecasting errors can occur due to factors such as changes in market conditions, unforeseen events, or inaccurate data. Holding safety stock allows retailers to hedge against forecasting errors by ensuring that they have additional inventory on hand to cover any discrepancies between actual and projected demand.

Service Level Requirements: Retailers may have service level agreements or customer service targets that require them to maintain a certain level of product availability. Safety stock helps retailers achieve these service level targets by reducing the risk of stockouts and ensuring that products are readily available to meet customer needs.

The annual Economic Order quantity (EOQ) of 18 would meet the average daily demand in Quarters 1-3. If the retailer could get advance notice of the bulk purchases, this may negate the need to hold safety stock at the

recommended levels. Recalculating the EOQ for Quarter 4 based on increased demand = 21. An alternative to holding safety stock could be to calculate EOQ by quarter, receive advance notice and/or change service level for bulk purchases.

Question 2.

a) For minimising average tardiness, follow the Earliest Due Date (EDD) rule. This results in the following schedule $B_3A_6D_{11}E_{17}$ $C_{21.}$ Here, maximum lateness (for case C) is = 21-16=5.

For minimising average tardiness we need to follow the Modified Due Date (MDD) heuristic. At time t, MDD is the maximum of "the due date and the earliest time at which we can complete a job".

Time t=0

Case	B	A	D	E	C
Processing Time (weeks)	3	3	5	6	4
Earliest due date (week)	8	9	9	12	16
MDD	8	9	9	12	16

$B_3A_6D_{11}E_{17}$ C_{21}
Total tardiness: (11-9)+(17-12)+(21-6)=12, Av Tardiness = 12/5=2.4

b) Here we can consider the Moore's algorithm that minimises the number of tardy jobs. Schedule is already in EDD, so we remove the job with the longest processing time before the first tardy job, D, which can be either jobs A or B. The schedule becomes B_3D_8 $E_{14}C_{18.}$ Then we remove D, resulting in $B_3E_9C_{10,}$ all of which are on time.
Hence jobs A and D should be subcontracted. However, some students might notice that only subcontracting D would allow all jobs to be completed. Assuming capacity is in place, subcontracting only D is a better option.

c) Students will discuss outsourcing criteria, such as cost, core competencies, loss of specialised skills, loss of control over quality.

d) Students should follow a Theory of Constraints approach to identify the bottleneck processing step, which is Staff 3, and calculate earnings per time on the bottleneck, resulting in a priority ranking as follows:

	A	B	C	D	E	SUM
Staff 1	1	4	2	3	6	16
Staff 2	3	2	4	12	4	25
Staff 3	5	8	6	6	4	29
Profit per case	19	24.375	43.33333333	70.8333333	27.5	
Rank	5	4	2	1	3	

Based on this ranking, all cases can be undertaken, except A.

e) Consider adding capacity before demand has increased if:
• Demand is forecast to grow and past experience shows forecasts are reliable
• The time required to add capacity is sufficiently long that failure to do so will lead to a competitive disadvantage
• The cost of initial investment in capacity is lower compared to the benefit of additional sales
• Delivery time is a key order winner
Consider waiting to add capacity until after demand has increased if:
• Delivery time is not a key order winner and customers would wait to buy rather than switch (e.g. for products with some degree of monopoly such as specialised law)
• Cost of adding capacity is too great to justify until firm orders are placed
• Capacity can be added rapidly, so that all risk can be eliminated from the investment decision. This is crucial in young businesses, where cash flow is critical. Production should be designed so that capacity can be increased or decreased rapidly – this is difficult in highly specialised service businesses such as law.

Section B

Question 3.

a.

i. The most appropriate approach to take is work sampling, which is a statistical technique for determining the proportions of time spent by subjects in various defined categories of activity. For statistical accuracy, observations must be taken at random times, and the period of the study must be representative of the types of activities performed by the subjects.

Work sampling is appropriate because:

- Sufficient time is available to perform the study
  - Several weeks usually required for a work sampling study
- Multiple subjects
  - Work sampling suited to studies involving more than one subject
- Nonrepetitive work cycles

- o Jobs consist of various tasks rather than a single repetitive task

The steps involved in a work sampling study are:

- Define the problem.
- Determine the required accuracy and required confidence limits.
    - Make a preliminary estimate of the % occurrence of the desired activity. Usually done by carrying out a pilot study.
    - Design the main study. Find number of observations required, design sampling plan with randomised times, determine number of observers.
        - o Carry out study according to above plan, summarising the data at the end of each shift

ii.The disadvantages/limitations of work sampling are:

- Not as accurate for setting time standards as other work measurement techniques
- Usually not practical to study a single subject
- Work sampling provides less detailed information about work elements than DTS or PMTS
- Since work sampling deals with multiple subjects, individual differences will be missed
- Workers may be suspicious because they do not understand the statistical basis of work sampling
- Behavior of subjects may be influenced by the act of observing them

b. (i) Normal time = average time * (PR/100) = 5.5 * (110/100) = 6.05 seconds
Standard time = normal time * (1 + AF/100) = 6.05 * (1 + 15/100) = 6.958 seconds

(ii) The required number of observations is given by the formula:

$$n=\left(\frac{t_{1-\frac{\alpha}{2}/n-1}\, s}{k\bar{x}}\right)^2$$

$$t_{1-\frac{\alpha}{2}/n-1}\,t_{n-1}^{1-\frac{\alpha}{2}}$$

= 2.262 for 9 degrees of freedom (10 observations)

s= 0.63 (standard deviation)
k= 0.05 (5% error)
$\bar{x}$= 5.5 (mean of the current observations)

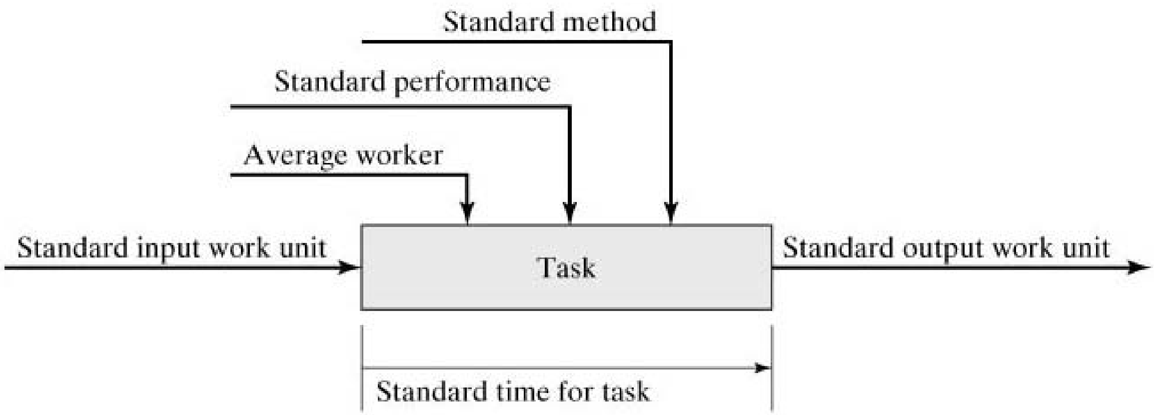
Substituting, we get n=26.7. Therefore, an additional 17 observations need to be made.

c. Variability in observed times can be caused by several factors:

- Differences in operator skill and experience: Different operators may have varying levels of familiarity with the task, leading to differences in execution time. Method to reduce variability: Standardize the task by training all operators to a consistent level of proficiency and by having a standard SOP before conducting the time study.

- Environmental conditions: Changes in factors such as lighting, temperature, or noise may impact the operator's performance and affect the observed times. Method to reduce variability: Conduct the time study in a controlled environment to minimize external influences.
- Task Complexity: Tasks with multiple steps can lead to inconsistent execution. Providing clear instructions and dividing tasks into sub-tasks can help reduce this variability.
- Fatigue: Fatigue affects performance over time. Scheduling regular breaks helps maintain consistent operator performance.
- Learning Curve Effect: Initial variability can be caused by operators still learning the task. Allowing a warm-up period ensures that operators have stabilized their performance.
- Equipment Variability: Variations in tools or machines used can lead to inconsistent times. Standardizing equipment ensures consistency in task execution.
- Measurement Error: Inaccurate timing can introduce variability. Using automated timing tools can eliminate human errors in recording observed times.

The factors that must be standardised before a time standard can be set are shown in the figure below:



Question 4.

a. The early stages of a product's life when a customer first begins to use the product is characterized by a high but rapidly decreasing failure rate. The high failure rate during this "burn-in" period accounts for parts with slight manufacturing defects not found during manufacture's testing. This decreasing failure rate typically lasts several weeks to a few months.

Due to this high failure rate in the early stages of component life, burn-in has been widely accepted as a method of screening out failures before systems are used by the customer. Burn-in is a screening operation that combines appropriate electrical and thermal environments to approximate, in a shortened time span, the real early life of a component/system.

With insufficient burn-in, high initial failure rates cause high field repair costs. Yet, with excessive burn-in, the reduced failure rate will be at the cost of increased capital and recurring costs. Optimal burn-in is necessary for devices in critical applications and for nonrepairable systems. However, one may not need burn-in at all for inexpensive devices where repair is simple and the consequences of a failure are trivial.

[NOTE: A good response will provide a basic description of the burn-in. Excellent responses will discuss the following points: (i) linking burn-in with the concept of the bathtub curve and pointing to the early (infant mortality phase of the lifecycle); (ii) discusses the idea that the failure rate decreases during this phase, and hence the concept of burn-in is justified; (iii) mentions the various objectives for burn-in that leads to finding an optimal solution for burn-in time.

b.

i. The lifetime distribution model as provided in the question is a Weibull distribution. The reliability function is therefore given by

$$R(t)=e^{-(t/\eta)^{\beta}} \qquad \text{[Eq.1]} \; Rt=e-t/\eta\beta \qquad \text{[Eq.1]}$$

Most candidates are unlikely to remember the formula, and would need to derive this by integrating the failure function to calculate the cumulative failure probability

$$F(t) \, Ft$$

and using this to derive the reliability function:

$$R(t)=1-F(t) \, Rt=1-F(t)$$

The effect of accumulated operating time T_0 on the probability that a device can survive for an additional time t:

$$R(t|T_0)=P(t'>T_0+t|t'>T_0)$$

where $t'=T_0+t$ is the time elapsed at failure since the device was new.

Using conditional probability, we can write $R(t|T_0)=R(T_0+t)R(T_0)$

[Excellent responses would include a derivation/explanation of the above equation]

Applying the Weibull reliability function to this, we get

$$R(t|t_b)=R(t_b+t)R(t_b)=e^{-((t+t_b)/\eta)^{\beta}}e^{-(t_b/\eta)^{\beta}} \; \text{[Eq.2]}$$

ii.The reliability of the product at 2 years without burn-in is given by:

$$R(t=2\times12)=e^{-(2\times12/10\times12)^{0.4}}=0.539$$

The reliability of the product at 2 years with a burn-in of one month is given by (using Eq. 2)

$$R(t=24|t_b=1)=0.679=0.679$$

This gives an increase of 25.9%.

iii.The parameters of the problem are as follows:

C_bCb= cost per unit time per item for burn-in test = £1
C_fCf= cost per failure during burn-in = £10
C_oCo= cost per failure when operational = £100
$t_b$tb= length of burn-in testing = 1 month
tt= warranty period = 24 months
$R(t_b)$= Probability that a product survives the burn-in = $e^{-(t_b/\eta)^{\beta}}$= 0.788
$R(t|t_b)$= Probability that the product survives until the end of warranty period with burn-in = 0.679 (from Part ii)
$R(t)$Rt= Probability that the product survives until the end of warranty period without burn-in = 0.539 (from Part ii)
We can find out whether the approach taken by the company is cost-effective by comparing the expected costs with and without burn-in.

The cost incurred by the company with burn-in is the sum of expected costs incurred by the burn-in process and the expected warranty costs involved due to product failures during operation.

Expected cost per unit incl. burn-in =

$E[C]=C_bt_b+C_f[1-R(t_b)]+C_oR(t_b)[1-R(t|t_b)]$EC=Cbtb+Cf1−Rtb+CoRtb1−Rt|tb
= £28.40

The cost incurred by the company without burn-in is simply the expected warranty cost.

Expected cost per unit without burn-in = $C_o[1-R(t)]$Co1−Rt= £46.10

Since it costs the company more in warranty cost without the burn-in, it seems that the current approach taken by the company is cost-effective. However, this might not be the optimal burn-in time. Ideally the expression for the expected cost per unit including burn-in should be used to identify the optimal burn-in time. This could lead to further reduction in expected costs.