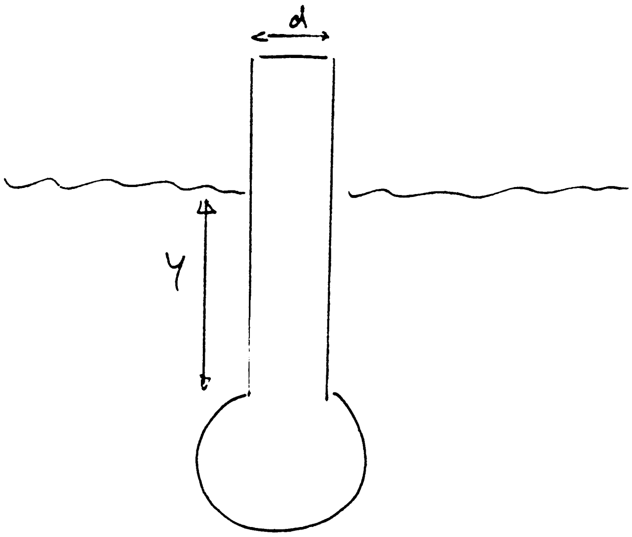
ENGINEERING TRIPOS PART IB

Solutions to Paper 4 1997

1)



a) Volume of float submerged is $V + \frac{\pi d^2 Y}{4}$

Hence upthrust $= \rho g \left[V + \frac{\pi d^2 Y}{4} \right]$ (Archimedes)

Equilibrium: $\rho g \left[V + \frac{\pi d^2 Y}{4} \right] = Mg$, ie $Y = \frac{4}{\pi d^2} \cdot \frac{M - \rho V}{\rho}$

b) Consider downwards displacement ΔY.

Net force on float $= \rho g \cdot \frac{\pi d^2}{4} \Delta Y$ upwards

Hence equation of motion: $M \Delta \ddot{Y} = - \rho g \frac{\pi d^2}{4} \Delta Y$

$$\Delta \ddot{Y} + \frac{\rho g}{M} \cdot \frac{\pi d^2}{4} \Delta Y = 0 \quad \text{(SHM)}$$

giving $\omega_n^2 = \frac{\pi \rho g d^2}{4M}$, $f_n = \frac{\omega_n}{2\pi} = \frac{d}{4} \sqrt{\frac{\rho g}{\pi M}}$ in Hz

c) Possible mechanisms causing oscillations to decay:

Waves will be generated by the motion of the float, and will carry energy away.

Viscous drag will oppose the motion of the float, and dissipate mechanical energy.

2) a) Bernoulli's equation is $p + \frac{1}{2}\rho V^2 + \rho g z$ = constant along a streamline

(p – pressure, ρ – density, V – velocity, z – height above datum)

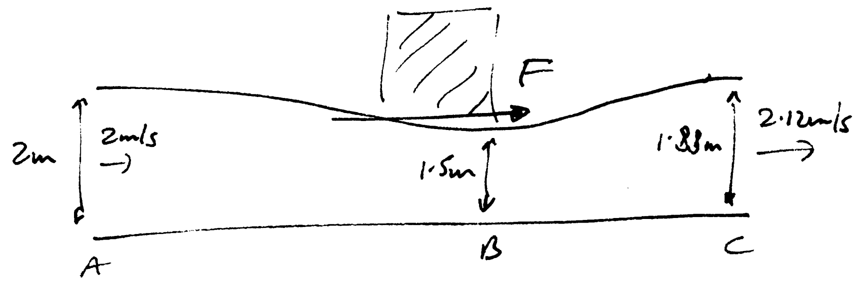
Valid for <u>steady</u>, <u>incompressible</u>, <u>inviscid</u> flow.

Terms: $\frac{1}{2}\rho V^2$ represents KE of fluid particles

$\rho g z$ —— " —— PE —— " ——

p represents work done on fluid particles by pressure gradient $\partial p / \partial s$

b)



(i) Bernoulli along <u>free surface</u> from $A \to B$ gives $\rho g d_A + \frac{1}{2}\rho V_A^2 = \rho g d_B + \frac{1}{2}\rho V_B^2$

ie $V_B = \sqrt{V_A^2 + 2g(d_A - d_B)} = 3.7162 = \underline{\underline{3.72 \text{ m/s}}}$

(ii) Continuity gives $50 . V_A d_A = (50 - 5w) V_B d_B$

$$w = 10\left[1 - \frac{V_A d_A}{V_B d_B}\right] = 2.8262 = \underline{\underline{2.82 \text{ m}}}$$

c) If F is the force ~~exerted~~ on the river by the bridge, then momentum gives

$$F + \frac{\rho g}{2}\left[d_A^2 - d_C^2\right].50 = \rho . 50 V_A d_A \left[V_C - V_A\right]$$

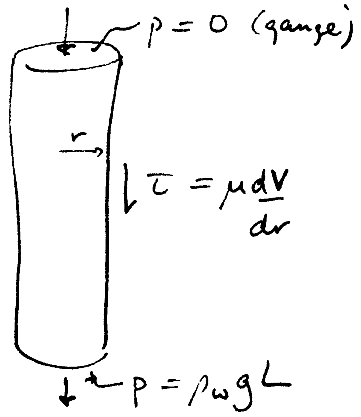
$$F = \underline{-90 \text{ kN}} \quad \text{(higher accuracy not justifiable)}.$$

Hence force exerted by river on bridge is $\underline{\underline{90 \text{ kN}}}$ in <u>downstream</u> direction

d) The loss in mechanical energy is due to the dissipative viscous forces acting as the high speed flows between the piers mix with the low speed (almost stagnant) flows behind the piers to form the uniform flow at C. The pressure gradient from $B \to C$ below the free surface also reduces the mechanical energy on the underwater streamlines, →

2d (cont'd) : but this loss is recoverable (as it is associated with p). 4/3

3) a) Force equilibrium on the cylinder shown here gives :



$$\mu \frac{dV}{dr} \cdot 2\pi r L + \rho_s g \cdot \pi r^2 L - \rho_w g L \cdot \pi r^2 = 0$$

shear stress sludge weight end pressures

ie $$\frac{dV}{dr} = -\frac{gr}{2\mu}(\rho_s - \rho_w)$$

b) Volumetric flow rate: $$Q = \int_0^R V(r) \cdot 2\pi r\, dr$$

From (a) $$V = -\frac{gr^2}{4\mu}(\rho_s - \rho_w) + c$$

$V = 0$ @ $r = R$ $\Rightarrow$ $$V = \frac{g(\rho_s - \rho_w)}{4\mu}\left[R^2 - r^2\right]$$

Hence $$Q = \frac{g(\rho_s - \rho_w)}{4\mu} \cdot 2\pi \int_0^R \left[R^2 r - r^3\right] dr$$

$$= \frac{\pi g(\rho_s - \rho_w)}{2\mu}\left[\frac{R^4}{2} - \frac{R^4}{4}\right] = \frac{\pi g R^4 (\rho_s - \rho_w)}{8\mu}$$

c) $$Q = \frac{\pi g R^4 \rho_w}{8\mu}\left[\rho_s/\rho_w - 1\right]$$

Q has dimensions m³/s, so can non-dimensionalise on $\sqrt{gR^5}$

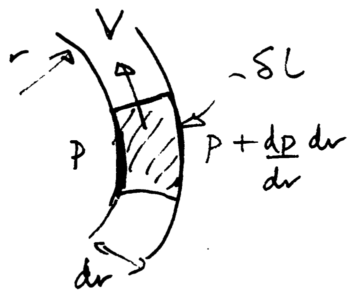
$$\frac{Q}{\sqrt{gR^5}} = \frac{\pi}{8}\boxed{\frac{g^{1/2} R^{3/2} \rho_w}{\mu}}\left[\frac{\rho_s}{\rho_w} - 1\right]$$

↑ this group must be constant.

Alternatively, non-dimensionalise Q on $\mu R/\rho_w$: $$\frac{Q}{(\mu R/\rho_w)} = \frac{\pi}{8}\left(\frac{g^{1/2} R^{3/2} \rho_w}{\mu}\right)^2\left(\frac{\rho_s}{\rho_w} - 1\right)$$

4). a) Flow with semi-circular streamlines:

Newton II for fluid element gives



$$\left[p - \left(p + \frac{dp}{dr}dr\right)\right]\delta l = -\rho(\delta l\, dr)\frac{V^2}{r} \quad \text{(centripetal accel}^{\underline{n}}\text{)}$$

ie $$\frac{dp}{dr} = \frac{\rho V^2}{r} \qquad \text{— (i)}$$

b) Flow is steady, incompressible and inviscid → can apply Bernoulli along s'lines:

$$p_a + \frac{1}{2}\rho V_0^2 = p + \frac{1}{2}\rho V^2$$

Differentiate: $$\frac{dp}{dr} = -\rho V\frac{dV}{dr}$$

Substitute into (i): $$\frac{VdV}{dr} + \frac{V^2}{r} = 0$$

$$\frac{dV}{V} + \frac{dr}{r} = 0$$

$$rV = \text{const.}$$

But at $r = r_1$, we know that $p = p_a$, and hence $V = V_0$, so

$$V(r) = \frac{V_0 r_1}{r}$$

c) Continuity gives

$$V_0 h = \int_{r_1}^{r_2} V(r)\,dr = V_0 r_1 \ln(r_2/r_1)$$

ie $$h = r_1 \ln(r_2/r_1)$$

d) Pressure at $r = r_2$ from Bernoulli:

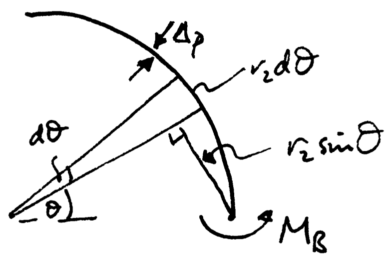
$$p_2 = p_a + \frac{1}{2}\rho\left(V_0^2 - V_2^2\right)$$

$$= p_a + \frac{1}{2}\rho V_0^2\left[1 - r_1^2/r_2^2\right]$$

Hence pressure difference across thrust reverser:

$$\underline{\underline{\Delta p = \frac{1}{2}\rho V_0^2\left[1 - r_1^2/r_2^2\right]}}$$

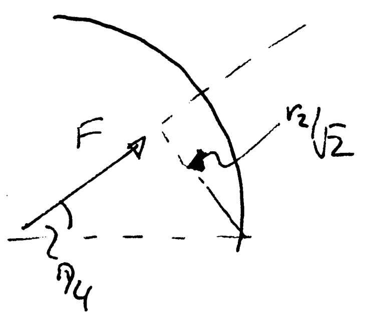
Bending moment:



$$M_B = \int_0^{\pi/2} \Delta p.\left(r_2\, d\theta\right).\, r_2 \sin\theta$$

$$= r_2^2 \Delta p \int_0^{\pi/2} \sin\theta\, d\theta = \underline{\underline{\frac{1}{2}\rho V_0^2\left[r_2^2 - r_1^2\right]}}$$

Alternatively use resultant force acting at centre of pressure:



$$F = \Delta p .\ 2 .\ r_2/\sqrt{2} = \sqrt{2}\, r_2 \Delta p$$

$$M_B = F .\ r_2/\sqrt{2}$$

$$= \underline{\underline{r_2^2 \Delta p}} \quad \text{as before.}$$

5). a) SFEE for adiabatic flow, no shaft work:

$$h + \frac{1}{2}V^2 = \text{constant}, \quad \text{or} \quad T + \frac{V^2}{2c_p} = \text{const}$$

for perfect gas

Hence $$T = 216 + \frac{718^2}{2.1010}$$

$$= \underline{\underline{471.2\,K}}$$

b) Combustion: $$C_8H_{18} + \frac{25}{2}\left(O_2 + \frac{79}{21}N_2^*\right) \rightarrow 8CO_2 + 9H_2O + \frac{25}{2}\frac{79}{21}N_2^*$$

Work with 1kg C_8H_{18}, then constituents of reactants/products are

	C_8H_{18}	Air	CO_2	H_2O	N_2^*
kmols :	$1/114$	$\frac{25}{2}.\frac{100}{21}.\frac{1}{114} =$ 0.5221	0.07018	0.07895	0.4125
kgs :	1	15.14	3.088	1.421	11.61

SFEE gives $\Delta H = 0$

$$\text{ie } (H_p - H_{po}) + (H_{po} - H_{ro}) + (H_{ro} - H_r) = 0$$

$$H_p - H_{po} = (H_r - H_{ro}) + (H_{ro} - H_{po})$$

$$= 0.5221 \times \left[13.72 - 8.64\right] \times 10^6$$ ← $H_r - H_{ro}$ (air)

$$+ 44430 \times 10^3$$ ← $H_{ro} - H_{po}$

$$= \underline{\underline{47.08 \times 10^6\,J}}$$

Now assume final temp. of 2500K, and calculate corresponding value

of $H_P - H_{P0} = \frac{8}{114}\left[131.61 - 9.37\right] + \frac{9}{114}\left[108.94 - 9.90\right] + 0.4125\left[83.01 - 8.67\right]$

$= \underline{\underline{47.06 \text{ MJ}}}$

Hence SFEE is satisfied, and temperature of products leaving combustion chamber is 2500K.

c) To calculate γ for mixture, need c_p, c_v for mixture.

$c_p = \sum_i m_i c_{p_i}$, where m_i is mass fraction of species i.

ie $c_p = \frac{1}{3.088 + 1.421 + 11.61}\left[3.088 \times 1.40 + 1.421 \times 2.95 + 11.61 \times 1.30\right]$

$= 1.4646$ kJ/kg

Similarly $c_v = 1.1788$ kJ/kg

Hence $\gamma = c_p/c_v = 1.2425$

and $T_{exit} = \frac{2 \times 2500}{\gamma + 1} = 2229.7$

$= \underline{\underline{2230K}}$

6) a) F_{12} is the proportion of the radiation leaving body 1 that is incident on body 2. (It is assumed that body 1 radiates uniformly in all directions.)

Consider radiative heat exchange between two black bodies:

radiation leaving body 1 is $A_1 E_{b1}$ $\quad (E_{b1} = \sigma T_1^4)$

radiation leaving body 2 is $A_2 E_{b2}$

Hence net heat transfer $q_{12} = A_1 F_{12} E_{b1} - A_2 F_{21} E_{b2}$

Now if the bodies are in thermal equilibrium then $T_1 = T_2$ (ie $E_{b1} = E_{b2}$)

and $q_{12} = [A_1 F_{12} - A_2 F_{21}] E_{b1} = 0$ (no net heat transfer)

Hence $\underline{\underline{A_1 F_{12} = A_2 F_{21}}}$, a result which (since it is purely geometric) also holds for grey bodies.

b) Use electrical analogy: $E_b - J = \dfrac{1-\varepsilon}{A\varepsilon} q$

and $J_1 - J_2 = \dfrac{1}{A_1 F_{12}} q_{12}$

Subscripts: h : heater, $\dfrac{1-\varepsilon}{A\varepsilon} = 2.5$

w : workpiece, $\dfrac{1-\varepsilon}{A\varepsilon} = 10$

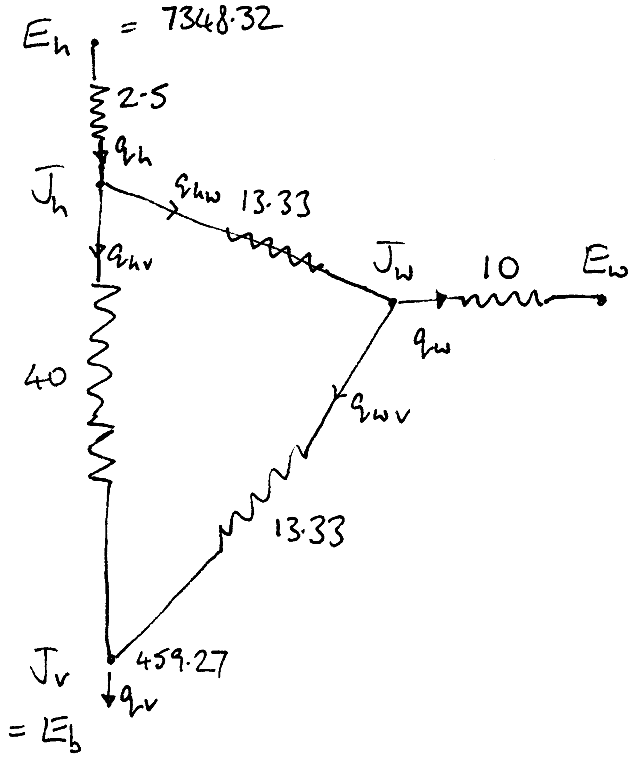
v : vessel $\dfrac{1-\varepsilon}{A\varepsilon} = 0$ (A large)

Heater → workpiece : $\dfrac{1}{A_h F_{hw}} = 13.33$

→ surroundings : $F_{hv} = 1 - F_{hw} = 0.25$ $\quad \dfrac{1}{A_h F_{hv}} = 40$

Workpiece → surroundings: $F_{wh} = \dfrac{A_h F_{hw}}{A_w} = 0.5$

$F_{wv} = 1 - F_{wh} = 0.5$ $\quad \dfrac{1}{A_w F_{wv}} = 13.33$



Insulated workpiece : $q_w = 0$, $J_w = E_w$.

Resistance of network : $R = 2{\cdot}5 + \left[\frac{1}{40} + \frac{1}{26{\cdot}66}\right]^{-1} = 18{\cdot}50$

Hence $q_h = q_v = \dfrac{7348{\cdot}32 - 459{\cdot}27}{18{\cdot}50} = 372{\cdot}43 = \underline{\underline{372\,W}}$

$J_h = 7348{\cdot}32 - 2{\cdot}5 \times 372{\cdot}43 = 6417{\cdot}25$

$\Rightarrow J_w (= E_w) = \dfrac{459{\cdot}27 + 6417{\cdot}25}{2} = 3438{\cdot}26$

$T_w = 496{\cdot}24 = \underline{\underline{496\,K}}$

(c) Now $q_w = 100\,W$ for the workpiece to remain in equilibrium while losing 100W through its rear insulation.

$J_w = 3438{\cdot}26 + 10 \times 100 = 4438{\cdot}26 \Rightarrow q_{wv} = \dfrac{4438{\cdot}26 - 459{\cdot}27}{13{\cdot}33} = 298{\cdot}$

$q_{hw} = 100 + q_{wv} = 398{\cdot}50 \Rightarrow J_h = 9750{\cdot}27 \Rightarrow q_{hv} = 232{\cdot}27$

$E_h = J_h + 2{\cdot}5\,[q_{hv} + q_{hw}] = 11327{\cdot}21$, ie $T_h = \underline{\underline{669\,K}}$