ENGINEERING TRIPOS PART IB

Thursday 4 June 1998 9 to 11

Paper 5

ELECTRICAL ENGINEERING

Answer not more than **four** *questions.*

Answer at least **one** *question from each section.*

The **approximate** *number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

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SECTION A

Answer at least **one** *question from this section.*

1 (a) Small variations in collector current, I_C, and base-emitter voltage, V_{BE}, about a d.c. operating point for a bi-polar transistor may be written as shown below: [3]

$$\delta I_C = \frac{\partial I_C}{\partial I_B}\delta I_B + \frac{\partial I_C}{\partial V_{CE}}\delta V_{CE}$$

$$\delta V_{BE} = \frac{\partial V_{BE}}{\partial I_B}\delta I_B + \frac{\partial V_{BE}}{\partial V_{CE}}\delta V_{CE}$$

Write these equations in terms of small-signal quantities v_{be}, v_{ce}, i_b, i_c and small-signal transistor parameters h_{ie}, h_{fe}, h_{oe} and h_{re}. Hence derive the small-signal equivalent circuit for the bi-polar transistor.

(b) The transistor in the common-emitter amplifier of Fig. 1 is to be biased so that $I_C = 25$ mA, $V_{CE} = 10$ V and $V_{BE} = 0.7$ V. The d.c. current gain at this operating point is 200. Find suitable values for R_C and R_B. [3]

(c) Draw a small-signal equivalent circuit for the amplifier, valid at mid-band frequencies. Determine the value of load resistance, R_L, which maximises the load power, and with this value for R_L, determine the mid-band voltage gain. [5]

The small-signal parameters of the transistor are $h_{ie} = 800\ \Omega$, $h_{fe} = 200$, $h_{oe} = 0$ S, $h_{re} = 0$.

(d) Explain why capacitors C_1 and C_2 reduce the gain of the amplifier at low frequencies. By ignoring the shunting effect of R_B on h_{ie}, find an expression for the gain, including the effect of both of these capacitors. [5]

(e) For $C_1 = C_2 = 20\ \mu$F, find the half-power frequency of the amplifier. [4]

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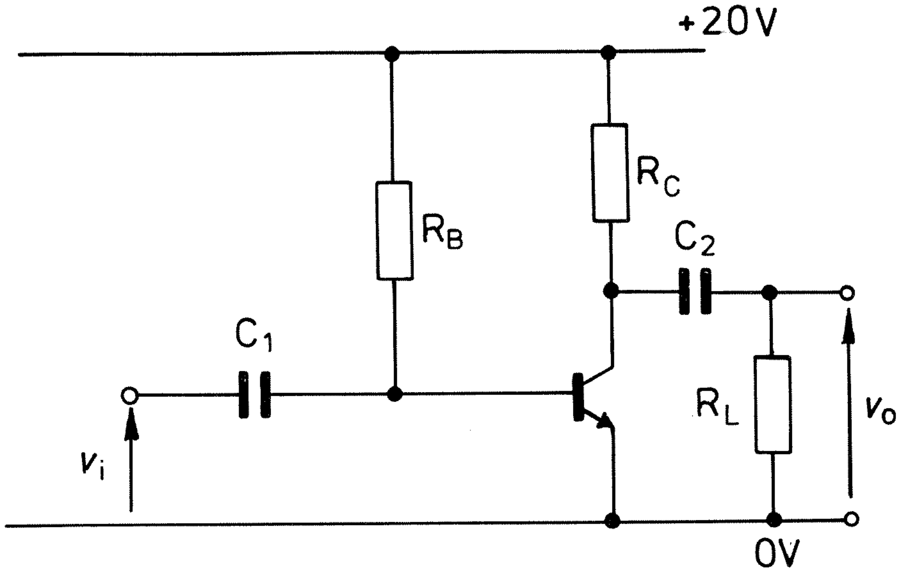

Fig. 1

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2 (a) The differential amplifier shown in Fig. 2a is to be used to amplify the voltage signal from a thermocouple, typically of the order 1 mV. Because of mains interference there is a significant unwanted common-mode signal, of the order 10 mV. Briefly explain the terms differential signal and common-mode signal, and explain why a differential amplifier is a good choice for this application. [4]

(b) Draw a small-signal circuit for the differential amplifier shown in Fig. 2a from which the common-mode gain may be calculated. You may ignore the small-signal transistor parameters h_{oe} and h_{re}. Hence find an expression for the common-mode gain, defined as v_a/v_1. Find also an expression for the differential gain, defined as $(v_a-v_b)/(v_1-v_2)$. [7]

Hence show that the common-mode rejection ratio is:

$$CMRR = \frac{h_{ie} + 2(h_{fe} + 1)R_T}{h_{ie}}$$

If $h_{ie} = 1\ \text{k}\Omega$ and $h_{fe} = 200$, find values for R_C and R_T to give a differential gain of 1000, and a common-mode rejection ratio of 2000.

(c) Local negative feedback, in the form of the resistor R_E as shown in the circuit of Fig. 2b, is used to improve the performance of the amplifier. Find an expression for the differential gain of the modified amplifier, and an expression for the factor by which this feedback has reduced the differential gain. [5]

(d) Find the value for R_E such that the input resistance of the modified amplifier is increased by a factor of 10 when compared with that of the amplifier of Fig. 2a. [4]

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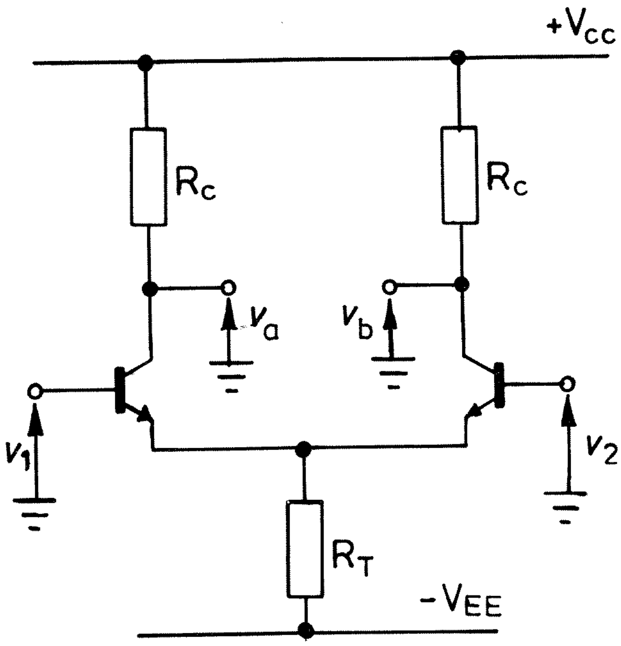

Fig. 2a

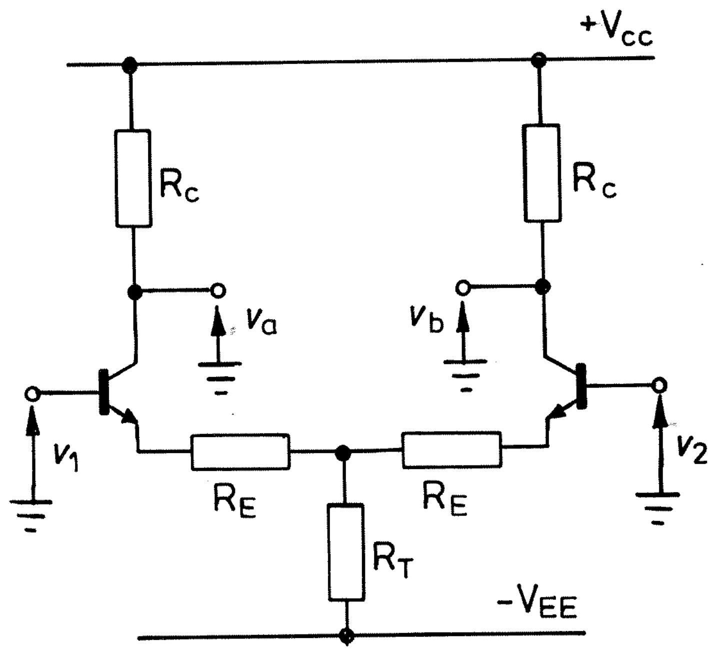

Fig. 2b

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SECTION B

Answer at least **one** *question from this section.*

3 (a) Give two reasons why electrical power is mainly generated as three-phase alternating voltages. [3]

(b) A 415 V, 50 Hz, three-phase supply feeds 2 loads connected in parallel as shown in Fig. 3. Load 1 is delta connected and consists of a $10\ \Omega$ resistor in parallel with a $10\ \Omega$ inductive reactance. Load 2 is star connected and consists of a $10\ \Omega$ resistor in series with a $5\ \Omega$ capacitive reactance. Calculate: [11]

(i) the power dissipated in each load;

(ii) the line current;

(iii) the power factor of the combined load.

(c) The power factor of the combined load can be altered by placing 3 identical star-connected capacitors in parallel with the load. [6]

Determine the value of capacitor required to correct the power factor to:

(i) unity;

(ii) 0.95 lagging.

For each capacitor calculate the percentage reduction in line current. Hence explain why it is not necessarily desirable to correct the power factor to unity.

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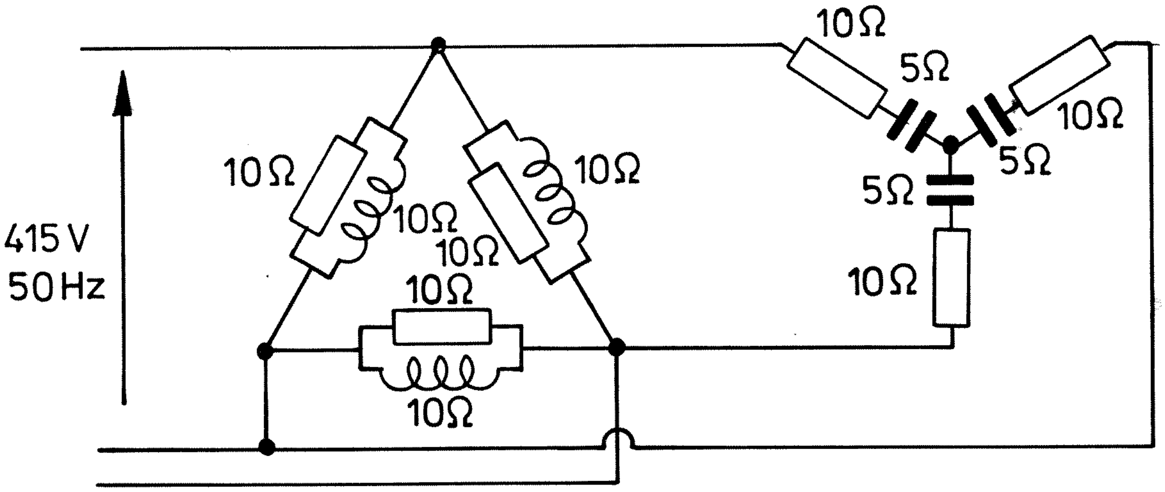

Fig. 3

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4 (a) Show how the operating chart for a three-phase synchronous generator can be obtained from the phasor diagram for the machine. [3]

(b) A 1000 MVA, 60kV synchronous generator is star-connected. It has a synchronous reactance of $5\ \Omega$ and negligible stator resistance. It is driven by a steam turbine which has a rated output power of 800 MW. The maximum excitation produces a generated line emf of 135 kV.

(i) Draw, to scale, an operating chart for the generator, indicating the various operating limits. Include a 10 % stability margin. [8]

(ii) Explain the physical significance of each of the operating limits. [4]

(iii) Calculate the range of power factors at which rated MVA can be delivered (you must *not* use a scaled drawing for this). [5]

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5 (a) Draw a typical torque/speed curve for a three-phase induction motor in the slip range $0 < s < 1$. [2]

(b) Figure 4 shows the per-phase equivalent circuit for an induction motor, for which the magnetising reactance and iron loss resistance can be ignored. Show that the output torque of the motor is given by: [6]

$$T = \frac{3V^2}{\left(R_1 + R_2'/s\right)^2 + \left(X_1 + X_2'\right)^2} \times \frac{R_2'}{s\omega_s}$$

(c) Using the maximum power transfer theorem, show that the maximum torque developed by the motor occurs at a slip given by: [5]

$$s_{max} = \frac{R_2'}{\sqrt{R_1^2 + \left(X_1 + X_2'\right)^2}}$$

(d) A wound-rotor, 3-phase, four-pole induction motor is star-connected to a 415V, 50 Hz supply. The parameters of the motor are: [3]

$R_1 = 1\ \Omega$, $R_2' = 1.4\ \Omega$, $X_1 = 2\ \Omega$, $X_2' = 2.4\ \Omega$

and it has a stator/rotor turns ratio of 2.

Calculate the peak torque developed by the motor and the speed of rotation at which it occurs.

(e) What extra resistance should be connected to the rotor circuit via slip rings to obtain maximum starting torque? [4]

Explain why extra resistance cannot be added to a cage induction motor in this manner and give an alternative method for increasing the rotor resistance at starting.

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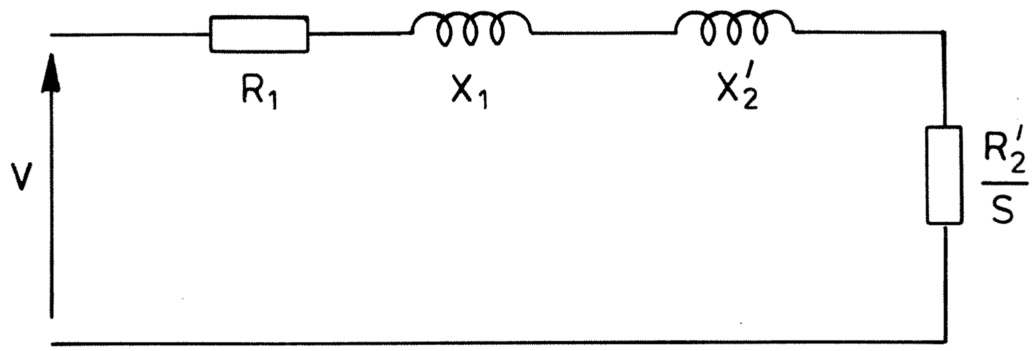

Fig. 4

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SECTION C

Answer at least **one** *question from this section.*

6 (a) A lossless transmission line has capacitance per unit length of $80\ \text{pFm}^{-1}$, and signals propagate along it at $2.5\times10^{8}\ \text{ms}^{-1}$. Find the inductance per unit length of the line and its characteristic impedance. Find also the wavelength of current and voltage waves when the line is connected to a 100 MHz a.c. voltage source. [3]

(b) Voltage and current vary with time (t) and distance along the line (x) for sinusoidal signals as: [8]

$$\overline{V}(x,t) = \overline{V}_f e^{j(\omega t-\beta x)} + \overline{V}_r e^{j(\omega t+\beta x)}$$

$$\overline{I}(x,t) = \overline{I}_f e^{j(\omega t-\beta x)} + \overline{I}_r e^{j(\omega t+\beta x)}$$

Briefly explain the physical meaning of these equations, and show that the current reflection coefficient for a transmission line of characteristic impedance Z_0 connected to a load of impedance $\overline{Z}_L$ is:

$$\overline{\rho}_I = \frac{\overline{I}_r}{\overline{I}_f} = \frac{Z_0 - \overline{Z}_L}{Z_0 + \overline{Z}_L}$$

Also write down an expression for the voltage reflection coefficient, $\overline{\rho}_V$.

(c) By substituting for $\overline{V}_r$ and $\overline{I}_r$ in terms of $\overline{V}_f$ and $\overline{I}_f$ respectively, show that the ratio of voltage to current (impedance) at any point on the line is: [5]

$$\frac{\overline{V}(x,t)}{\overline{I}(x,t)} = \overline{Z}(x) = Z_0 \frac{e^{-j\beta x} + \overline{\rho}_V e^{j\beta x}}{e^{-j\beta x} + \overline{\rho}_I e^{j\beta x}}$$

(d) The transmission line of part (a) is connected to a 100 MHz a.c. voltage source of output resistance $50\ \Omega$ at one end, and a load of impedance $(30+j40)\ \Omega$ at the other, as shown in Fig. 5. Find the impedance presented by the line at the signal source if the length of the line, l, is 2.5 m. Comment on your answer. [4]

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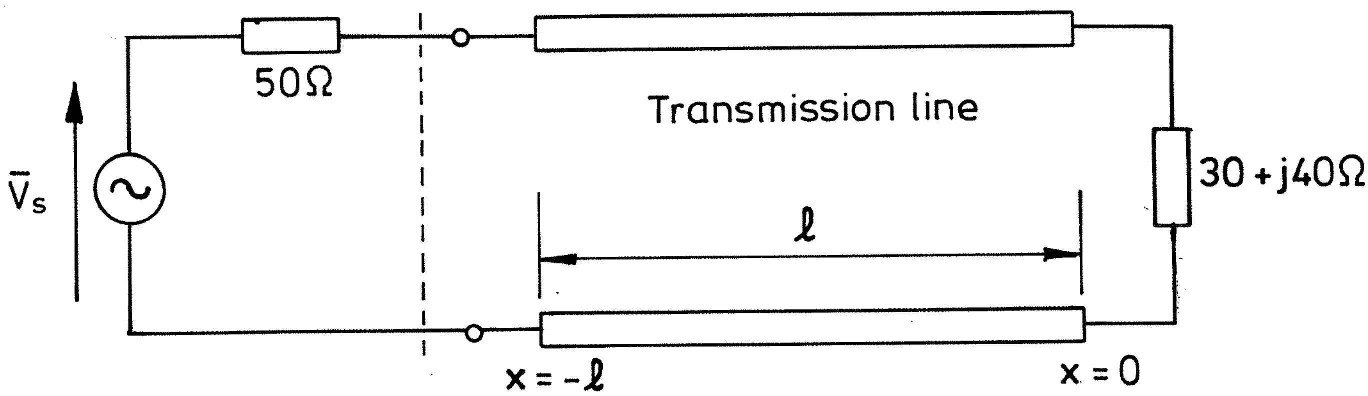

Fig. 5

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7. A plane electromagnetic wave travelling through free space has its electric field strength given by:

$$\boldsymbol{E} = \boldsymbol{u}_x E_0 e^{j(\omega t - \beta z)}$$

where $\boldsymbol{u}_x$ is a unit vector in the x direction.

(a) Find the condition which ω and β must satisfy for this plane electromagnetic wave to satisfy the governing wave equation given by: [4]

$$\frac{\partial^2 \boldsymbol{E}}{\partial z^2} - \varepsilon_0 \mu_0 \frac{\partial^2 \boldsymbol{E}}{\partial t^2} = 0$$

and hence show that the speed of propagation of the wave in free space is $1/\sqrt{\mu_0 \varepsilon_0}$.

(b) Show on a sketch the relationship between the direction of propagation and the electric and magnetic fields of this plane electromagnetic wave, and define the impedance of free space, η_0. Hence write down an expression for the corresponding magnetic field strength. Using the vector $\frac{1}{2}\boldsymbol{E} \times \boldsymbol{H}^*$ show that the power per unit area transmitted by the wave is $\frac{E_0^2}{2\eta_0}$. [7]

(c) The light from a 1 mW laser pen may be modelled as a plane electromagnetic wave with circular cross-section of diameter 1 mm travelling through free space. Determine the magnitude of its electric and magnetic field strengths. [5]

(d) The light impinges normally on a lossless material of permittivity $3\varepsilon_0$ and permeability μ_0. By analogy with transmission line theory, or otherwise, find the power per unit area which propagates into the material, and the magnitude of the electric and magnetic fields within the material. [4]

END OF PAPER