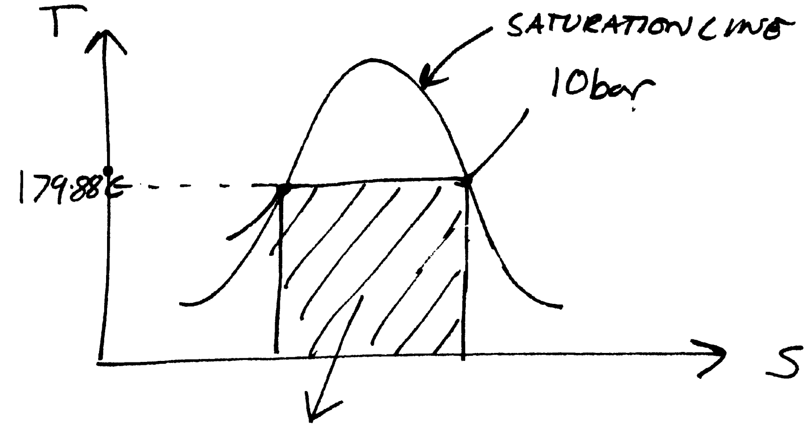
ENGINEERING TRIPOS, PART IB 2004

PAPER 4 - THERMOFLUID MECHANICS - CRIB

1 A



$Q = T\Delta S$

$S_f = 2.138e^3$ $\quad$ $S_g = 6.585e^3$ $\quad$ P19 CUED

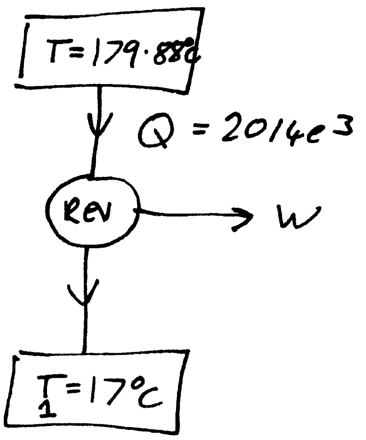
$Q = T\Delta S$ $\quad$ REVERSIBLE HEAT EXTRACTION

$$Q = (179.88 + 273) \times (-6.585e^3 + 2.138e^3)$$

$$= \underline{\underline{-2014\ \text{kJ}}}$$

$\boxed{5}$

B



$\Delta S_{WATER} + \Delta S_{IRON} = 0$

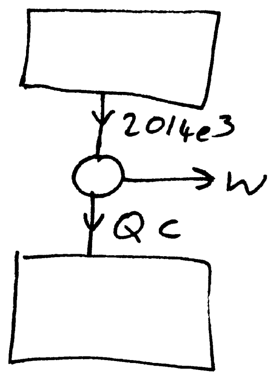
$$\frac{2014e^3}{(179.88+273)} = m C_v \ln \frac{T_2}{T_1}$$

$$\ln \frac{T_2}{T_1} = \frac{2014e^3}{(179.88+273) \times 70 \times 437}$$

$$T_2 = 335.4\ \text{K} = \underline{\underline{62.4\,°\text{C}}}$$

$\boxed{6}$

1 (B) CONT

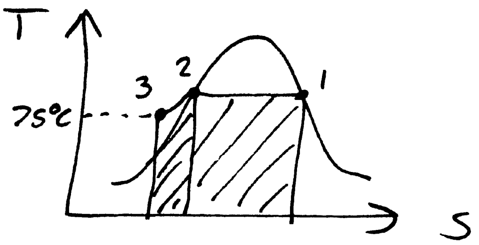


$$Q_c = MC\Delta T$$
$$= 70 \times 437 \times (62.4 - 17)$$
$$Q_c = 1388.8e3$$

$$W = (2014e^3 - 1388.8e^3) = 625.2e$$

$\boxed{2}$

(C) ASSUME FINAL T OF BOTH IS 75 °C

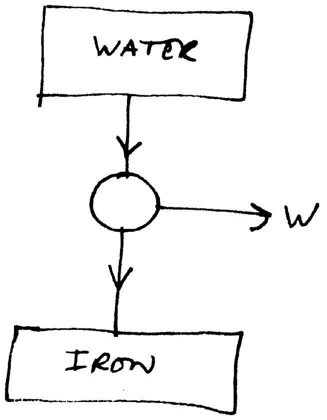


$$\Delta U = 0 - W$$

$$Q_{23} = (313.8 - 761.4) \; 10e^2 \times (0.001127 - 0.001$$
$$= -447.7 \text{ kJ}$$

$$Q_{TOT} = -(2014 + 447.7) = -2462e^3$$

$\boxed{2}$



$$\Delta S_{IRON} = mC_V \ln \frac{T_2}{T_1}$$
$$= 70 \times 437 \times \ln\left(\frac{75+273}{17+273}\right)$$
$$\Delta S_{IRON} = 5.57e^3$$

$$\Delta S_{WATER} = \dot{m}(\Delta s) = 1 \times (1.015 - 6.585)$$

$$= -5.57e^3$$

TO THREE SIGNIFICANT FIGURES ENTROPY OF WATER AN IRON ARE EQUAL
SO FINAL TEMPERATURE IS 75°C

2

$$Q_{IRON} = mC\Delta T = 70 \times 437 \times (75 - 17)$$

$$= 1774e^3$$

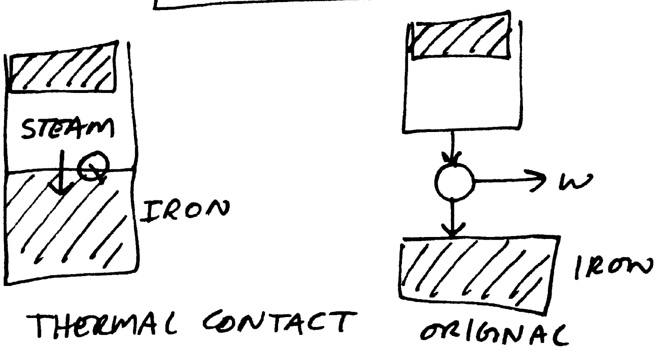
$$W = \text{[redacted]}\ Q_{TOT} - Q_{IRON} = 2462e^3 - 1774e^3$$

$$= 688e^3$$

1

$$\boxed{R = \frac{688}{1774} = 0.388}$$

NOT REQUIRED



THE IRON BLOCK WOULD BE AT A HIGHER TEMPERATURE
BECAUSE NO WORK WAS REMOVED

1

THE ENTROPY OF THE [redacted] SYSTEM WOULD HAVE GONE UP.
HEAT FLOW ACCROSS A FINITE TEMPERATURE DIFFERENCE.

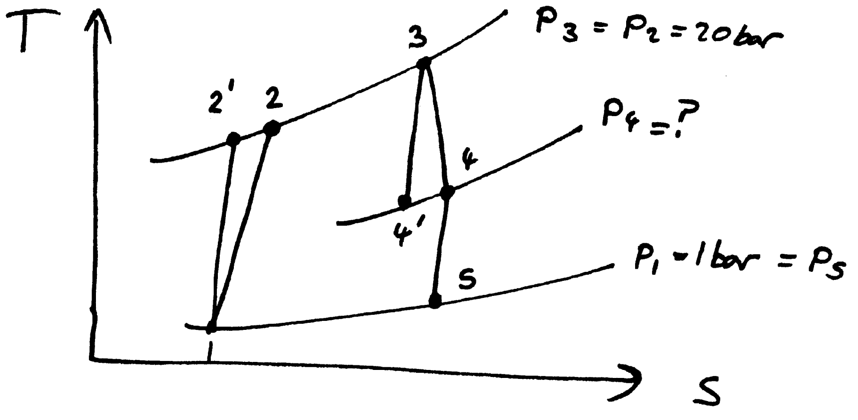
1

Examiners report 1B - Part A Thermodynamics

Question 1

The average mark on this question was 57%. Part A was correctly answered by nearly all candidates without a problem. Part B was a standard reversible heat engine mounted between one reservoir that varied in temperature and one that did not. 50% of candidates knew how to approach the question properly. The remainder tried to assume the temperature of both reservoirs remained constant. Part C was only answered well by about 15% of students. About 20% of the candidates who attempted part C did not realise that the process was reversible and assumed that no work was produced and that all the heat out of the water went into the iron block.

2 A



5

3

FOR COMPRESSOR

$$\frac{W_{ISENT}}{\dot{m}_{AIR}} = C_p T_1 \left[\left(\frac{P_2}{P_1} \right)^{\frac{\gamma - 1}{\gamma}} - 1 \right] = 1.01 \times 288 [$$

$$= 393.7 \text{ kJ/kg}$$

$$W_{REAL} = \frac{W_{IS}}{\eta_c} = \frac{393.7}{0.85} = 463.2$$

$$T_2 = 288 + \frac{463.2}{1.01} = 746.6\,K$$ 3

$$T_3 = T_2 + \frac{\Delta Q}{C_p} = 746.6 + \frac{450}{1.1} = 1192.1$$

$$T_3 = 919.1\,°C$$ 2

B $W_{TURB} = W_{COMP} = 463.2$

$$T_4 = 1192.1 - \frac{463.2}{1.01} = 733.5\,K$$ 2

$$\Delta h_{PROP} = \frac{1}{2} V^2 = C_p (T_4 - T_5)$$ ← TO FIND V

↑ REQUIRE

WE FIRST REQUIRE P_4 AND THUS REQUIRE T_4'

$$\eta_T C_p (T_3 - T_4') = C_p (T_3 - T_4)$$

② Ⓑ CONT

⑥

$$T_4' = T_3 - \frac{T_3 - T_4}{R_T} = 919.1+273 - \frac{(919.1+273-733.5}{0.87}$$

$$T_4' = 665.0\,K$$ $\boxed{2}$

$$\left(\frac{P_4}{P_3}\right)^{\frac{\gamma-1}{\gamma}} = \frac{T_4'}{T3}$$

$$P_4 = P_3 \times \left(\frac{T_4'}{T_3}\right)^{\frac{\gamma}{\gamma-1}} = 20 \times \left(\frac{665}{991+273}\right)^{\frac{1.4}{0.4}}$$

$$= 2.59\,bar$$

$$\Delta h_{PROP} = C_p T_4 \left(1 - \left(\frac{P_5}{P_4}\right)^{\frac{\gamma-1}{\gamma}}\right) = \frac{1}{2}V^2$$

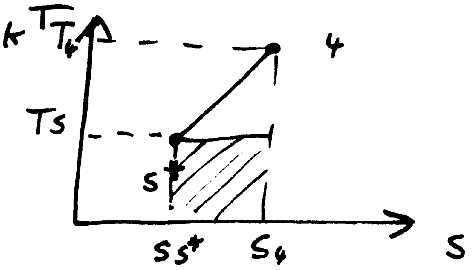
$$= 1.01e^3 \times 733.5 \left(1 - \left(\frac{1}{2.59}\right)^{\frac{\gamma-1}{\gamma}}\right)$$

$$\frac{1}{2}V^2 = 176.56e^3$$

$$V = \underline{\underline{594.2\,ms^{-1}}}$$ $\boxed{3}$

$$R_{TURB} = \frac{176.6\,e^3}{450e^3} = \underline{\underline{0.392}}$$

Ⓒ MAX WORK



S^* IS NON THEORETICAL S

$$MAX\ W_s = (h_4 - h_{5^*}) - T_0(S_4 - S_{5^*})$$

$$= C_p(733.5 - 288) - 288 \times \Delta S$$

2 c CONT

$$\Delta S = C_p \ln\left(\frac{T_4}{T_{5*}}\right) - R \ln\left(\frac{P_4}{P_{5*}}\right)$$

$$\Delta S = 1.01e^3 \ln\left(\frac{733.5}{288}\right) - 0.287e^3 \ln\left(\frac{2.59}{1}\right)$$

$$\Delta S = 670.8 \qquad \boxed{3}$$

$$W_{s\,MAX} = 1.01e^3 (733.5 - 288) - 288 \times 670.8$$

$$= 256.8e^3 \qquad \boxed{1}$$

$$R_{\text{CYCLE THEORETICAL}} = \frac{258.8}{450} = \underline{\underline{0.575}}$$

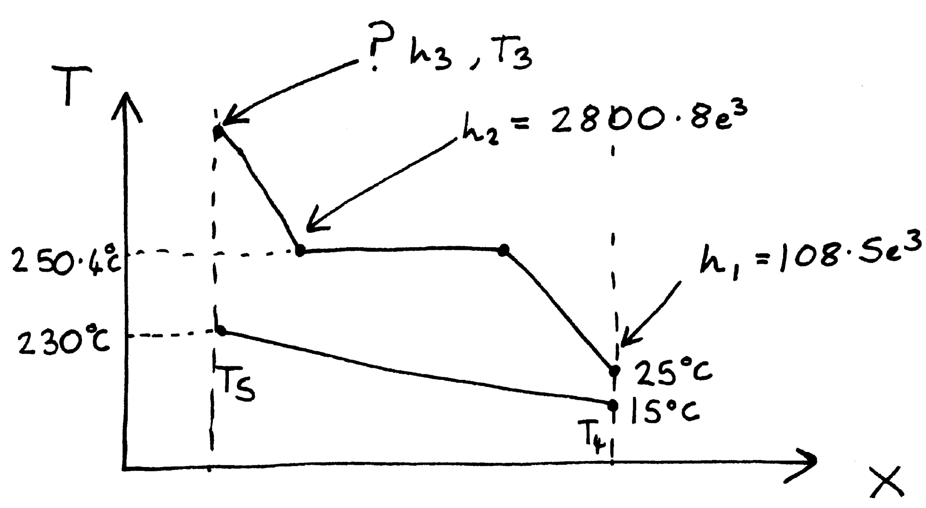
THE WEIGHT OF THE REVERSIBLE HEAT ENGINES WOULD BE TOO HIGH SO THE EXHAUST IN PRACTICE IS LEFT HOT (STRAIGHT TURBOJET) $\boxed{1}$

Question 2

The average mark for the question was 60%. Part A was answered correctly by nearly all candidates. Only ~50% of candidates sketched the T-S diagram perfectly. In part B a small number of candidates (~10%) assumed the turbine pressure ratio was 20 instead of matching compressor and turbine work. Most candidates managed to calculate the turbine exit temperature correctly. ~25% of candidates assumed that the nozzle expanded the gas to ambient temperature instead of pressure. ~30% of candidates completed section B in full. Part C was a simple application of the maximum work formula. Only about 10% of candidates made a reasonable attempt to this part of the question. Full marks were given for anyone who attempted to use the correct formula. It is clear that most of the candidates did not understanding the concept of maximum work.

3 (A)

8



ACCROSS WHOLE HEAT-EXCHANGER

$$\dot{m}_w \left(h_3 - 108.5e^3\right) = \dot{m}_c C_p \left(230-15\right)$$

$$h_3 = 108.5e^3 + 15 \times 0.93e^3 \left(230-15\right)$$

$$h_3 = 3.108e^6$$

P 21 CUED

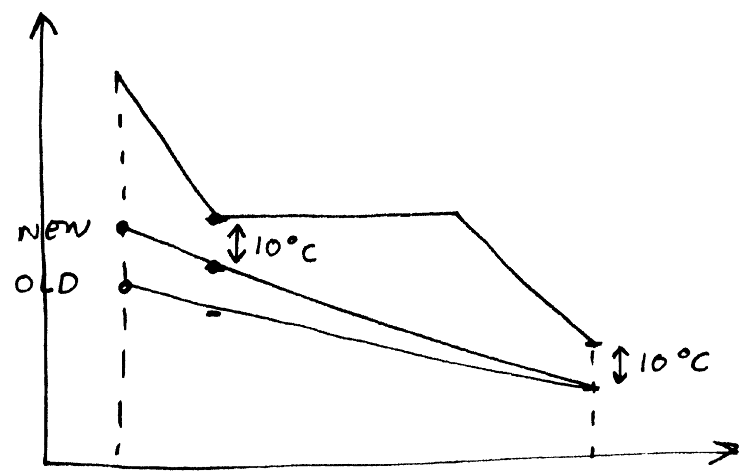
$$T_3 = \left(\frac{3108 - 3093.3}{3154.7 - 3093.3}\right) \times 25 + 350$$

$$T_3 = 356.0$$

6

STEAM TEMPERATURE AT INLET IS 356 °C

(B)



3 B

EQUATE HEAT FLOW BETWEEN TWO PINCH POINTS

$$\dot{m}_w\,(2800.8 - 108.5) = \dot{m}_c\,C_p\,(240.4 - 25)$$

$$\frac{\dot{m}_c}{\dot{m}_w} = \frac{(2800.8 - 108.5)}{0.93 \times (240.4 - 15)} = 12.8$$

CALCULATE FINAL TEMP OF CO_2 FOR NEW HEAT-EXCHANGER

$$\dot{m}_c\,C_p\,(T_5 - 15) = \dot{m}_w\,(3108e^3 - 108.5e^3)$$

$$T_5 = (3108e^3 - 108.5e^3) / (12.8 \times 0.93e^3) + 15$$

$$T_5 = 267.0\,^\circ C$$

$\boxed{7}$

C TOTAL ENTROPY FLOW $= \dot{m}_w \Delta S_w + \dot{m}_c \Delta S_c$

ENTROPY ↑ CO_2 ↑

$$\frac{\Delta S}{\dot{m}_w} = \Delta S_w + \frac{\dot{m}_c}{\dot{m}_w}\Delta S_c$$

(*) $\frac{\Delta S_{DIFF}}{\dot{m}_w}$ BETWEEN TWO HEAT EXCHANGERS $= \left(\cancel{\Delta S_w} + \frac{\dot{m}_c}{\dot{m}_w}\Delta S_c\right)_{NEW} - \left(\cancel{\Delta S_w} + \frac{\dot{m}_c}{\dot{m}_w}\right.$

$$\Delta S = C_p \ln\frac{T_2}{T_1} - R \ln\cancel{\frac{P_2}{P_1}} \quad \text{FOR } CO_2$$

0

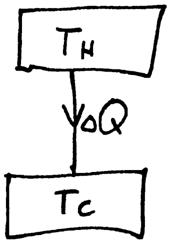
$$\Delta S_{c\,NEW} = 0.93e^3 \times \ln\left(\frac{(267.0 + 273)}{(15 + 273)}\right) = 584.6$$

$$\Delta S_{c\,OLD} = 0.93e^3 \times \ln\left(\frac{(230 + 273)}{(15 + 273)}\right) = 518.6$$

FROM (*)

$$\frac{\Delta S_{DIFF}}{\dot{m}_w} = 12.8 \times 584.6 - 15 \times 518.6 = -296.1$$

(C) CONT THE ENTROPY INCREASE OF THE HEAT-EXCHANGER IS DUE TO HEAT-TRANSFER ACCROSS A FINITE TEMPERATURE DIFFERENCE. THE FINITE ΔT HAS REDUCED WITH THE SECOND HEAT EXCHANGER

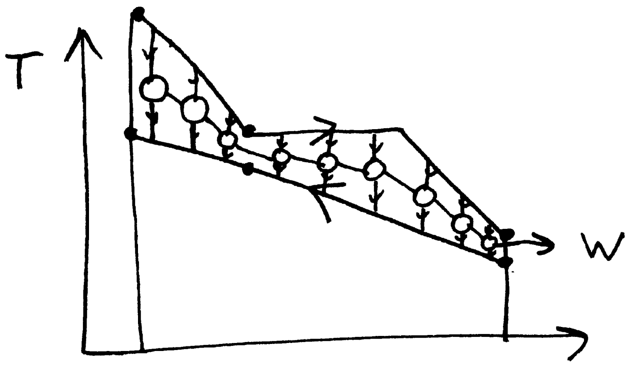


$$\Delta S = \frac{\Delta Q}{T_C} - \frac{\Delta Q}{T_H}$$

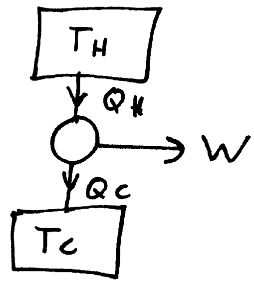
$T_H > T_C$ SO $\Delta S > 0$

$\boxed{5}$

(D)



TEMPERATURE DIFFERENCE BETWEEN STREAMS USED TO DRIVE INFITE SMALL HEAT PUMPS.



$$\Delta S = \frac{\Delta Q_H - W}{T_C} - \frac{\Delta Q_H}{T_H} = 0$$

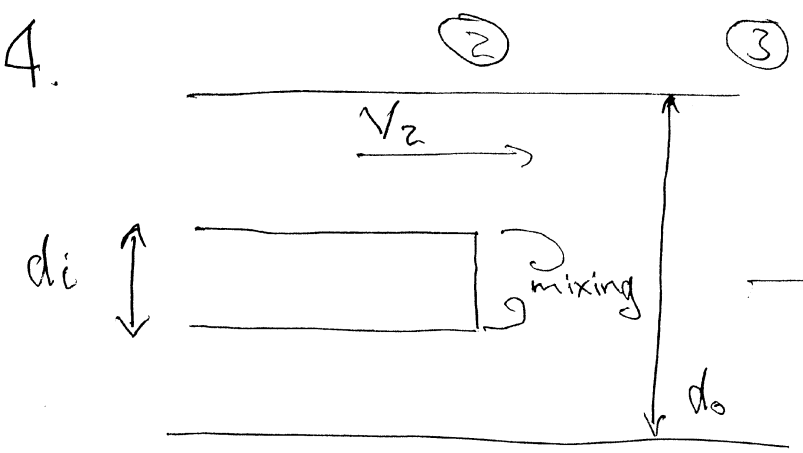
REVERSIBLE HEAT ENGINES

$\boxed{2}$

Question 3

The average mark on this question was 51%. The question was mainly chosen as a second question by candidates who scored badly on there first question. This lowered the average mark on the question. Of those who attempted the question only 25% scored highly. Part A of the question was easy. Unfortunately 30% of candidates tried to work out the enthalpy of water and steam using cp. 75% of candidates could not draw a diagram showing the temperature variation through the heat-exchanger. Part C of the question asked the candidates to calculate the entropy change. Most candidates who got to part C produced good solutions. Most candidates could answer the discussion points at the end of part C and part D.

4.



a) Continuity 2-3

$$\Rightarrow \frac{\pi (d_o^2 - d_i^2)}{4} V_2 = \frac{\pi d_o^2}{4} V_3$$

$$\Rightarrow V_3 = V_2 \left(1 - \left(\frac{d_i}{d_o} \right)^2 \right)$$

(b) There is mixing as the flow moves from the smaller to the larger area. Bernoulli's equation may NOT be used. This was the main source of error in this question

Momentum equation 2-3

$$(p_2 - p_3) \frac{\pi d_o^2}{4} = \underbrace{\rho \pi \left(\frac{d_o^2 - d_i^2}{4} \right) V_2}_{\dot{m}} \left(V_3 - V_2 \right)$$

$$\Rightarrow p_2 - p_3 = \rho \left[1 - \left(\frac{d_i}{d_o} \right)^2 \right] V_2^2 \left(1 - \left(\frac{d_i}{d_o} \right)^2 - 1 \right)$$

$$= - \rho V_2^2 \left[1 - \left(\frac{d_i}{d_o} \right)^2 \right] \left(\frac{d_i}{d_o} \right)^2$$

c) the change in flux increases the internal energy in the mixing process.

$$\Delta p_o = p_{o2} - p_{o3}$$
$$= (p_2 - p_3) + \frac{1}{2}\rho\left(V_2^2 - V_3^2\right)$$
$$= -\rho V_2^2\left[1-\left(\frac{d_i}{d_o}\right)^2\right]\left(\frac{d_i}{d_o}\right)^2 + \frac{1}{2}\rho V_2^2\left[1-\left(1-2\left(\frac{d_i}{d_o}\right)^2 + \left(\frac{d_i}{d_o}\right.\right.\right.$$
$$= \frac{1}{2}\rho V_2^2\left(\frac{d_i}{d_o}\right)^4$$

and the change in flux of mechanical energy

is $\dot{m} T \Delta s = -\frac{\dot{m}\,\Delta p_o}{\rho}$

$$= \frac{\pi}{8}\left(d_o^2 - d_i^2\right) V_2^3 \rho \left(\frac{d_i}{d_o}\right)^4$$

d) continuity ① – ② $\Rightarrow V_{x1} = V_{x2}$

momentum equation in axial direction ① – ② $\Rightarrow p_1 = p_2$

$$\therefore \quad \dot{m} T \Delta s = -\frac{\dot{m}\,\Delta p_o}{\rho} = \frac{V_2 \pi \left(d_o^2 - d_i^2\right)}{4}\,\frac{1}{2}\rho V_\theta^2$$

as $p_{o2} - p_{o3} = \frac{1}{2}\rho V_\theta^2$

5. Froude scaling $\Rightarrow \left(\frac{V}{\sqrt{gL}}\right)_m = \left(\frac{V}{\sqrt{gL}}\right)_{FS}$

$$\Rightarrow V_m = V_{FS} \sqrt{\frac{L_m}{L_{FS}}} = \underline{2.529 \, m/s}$$

b) for both $\left(\frac{\rho V L}{\mu}\right)_m = \left(\frac{\rho V L}{\mu}\right)_{FS}$

& $\left(\frac{V}{\sqrt{gL}}\right)_m = \left(\frac{V}{\sqrt{gL}}\right)_{FS}$

$$\Rightarrow \quad V_m = V_{FS} \frac{\rho_{FS}}{\rho_m} \frac{L_{FS}}{L_m} \frac{\mu_m}{\mu_{FS}}$$

& $$V_m = V_{FS} \sqrt{\frac{L_m}{L_{FS}}}$$

$$\therefore \quad 1 = \frac{\rho_{FS}}{\rho_m} \sqrt{\frac{L_{FS}}{L_m}} \frac{\mu_m}{\mu_{FS}} \text{ or } \frac{\nu_{FS}}{\nu_m} = \sqrt{\frac{L_{FS}}{L_m}}$$

not possible with known liquids and tank tests always use water

Wave drag correct. Re too low so inappropriate frictional drag (ie too high in the model

c) $Re = \frac{\rho V L}{\mu} = \frac{VL}{\nu}$ $\quad c_D = \frac{F_D}{\rho V^2 L^2}$

i) case ① $Re = 3.073 \times 10^6$ $\quad$ case ② $Re = 5.122 \times 10^{?}$

$c_D = 3.478 \times 10^{-4}$ $\quad$ $c_D = 3.140 \times 10^{-4}$

use $C_D = k Re^{-n}$
take logs

$$\Rightarrow \ln C_{D1} = \ln k - n \ln Re_1$$
$$\ln C_{D2} = \ln k - n \ln Re_2$$

$$\therefore \ln \frac{C_{D1}}{C_{D2}} = -n \ln \frac{Re_1}{Re_2}$$

$$\Rightarrow \underline{n = 0.2}$$

$$\& \quad \underline{k = 6.900 \times 10^{-3}}$$

d) $Re_{tank} = 3.793 \times 10^6$

$\Rightarrow C_{D\,tank\,friction} = 3.33 \times 10^{-4}$
$\Rightarrow F_{friction} = 4.798\,N$

$\Rightarrow \underline{F_{wave} = 60\%} \quad \underline{= 7.2\,N}$

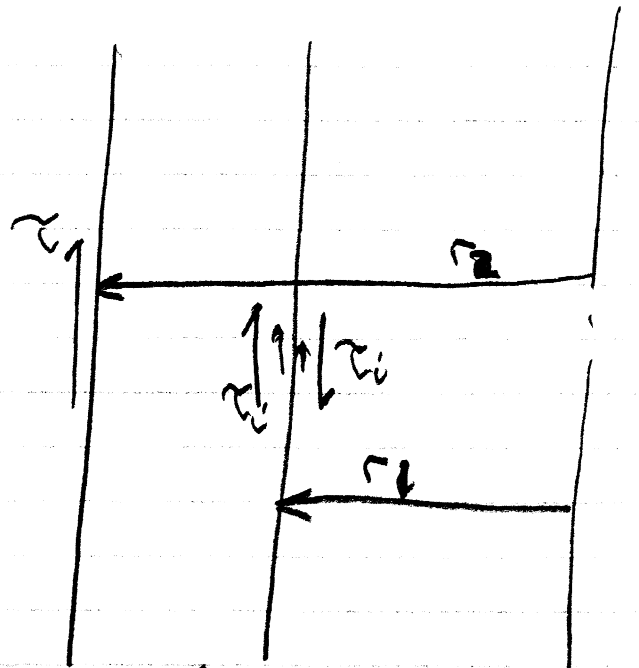
$C_{D\,wave} = 5.003 \times 10^{-4} \Rightarrow \underline{F_{wave\,FS} = 7.204\,kN}$

e) $C_{D\,friction} = k Re^{-n}$ $\qquad Re_{FS} = 120 \times 10^6$

$\Rightarrow C_{D\,friction} = 1.671 \times 10^{-4}$
$\Rightarrow F_{friction\,FS} = 2.406\,kN$

$\therefore \underline{F_{total} = 9.61\,kN}$

6.



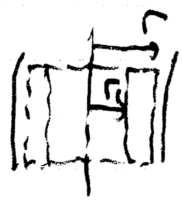
a) $\frac{\partial p}{\partial r} = \frac{\rho V^2}{r_c}$ but $r_c \to \infty \Rightarrow p$ const with r
continuity $\Rightarrow u$ const with z

b) mmtm equation in oil

$$\tau_i 2\pi r_1 \delta z - \frac{dp}{dz}\delta z \pi r_1^2 - \rho_o g \pi r_1^2 \delta z = 0$$

$$\Rightarrow \quad \tau_i = \left(\frac{dp}{dz} + \rho_o g\right)\frac{r_1}{2}$$

c) mmtm equation in water from $r_1 \to r$



$$-\tau_i 2\pi r_1 \delta z + \tau 2\pi r \delta z - \frac{dp}{dz}\delta z \pi (r^2 - r_1^2) - \rho g \pi (r^2 - r_1^2)\delta z = 0$$

$$\Rightarrow \quad \tau r = \frac{dp}{dz}\left(\frac{r^2 - r_1^2}{2}\right) + \rho g\left(\frac{r^2 - r_1^2}{2}\right) + \tau_i r_1$$

subst for τ_i

$$\Rightarrow \tau r = \frac{dp}{dz}\frac{r^2 - r_1^2}{2} + \rho g\left(\frac{r^2 - r_1^2}{2}\right) + \frac{dp}{dz}\frac{r_1^2}{2} + \rho_o g \frac{r_1^2}{2}$$

$$\Rightarrow \tau = \mu \frac{du}{dr} = \left(\frac{dp}{dz} + \rho g\right)\frac{r}{2} + (\rho_0 - \rho) g \frac{r_1^2}{2r}$$

$$\Rightarrow \mu u = \left(\frac{dp}{dz} + \rho g\right)\frac{r^2}{4} + (\rho_0 - \rho) g \frac{r_1^2}{2} \ln r + A$$

b.c. $u = 0$ @ $r = r_2$

$$\Rightarrow u = \left(\frac{dp}{dz} + \rho g\right)\frac{r^2 - r_2^2}{4\mu} + (\rho_0 - \rho)\frac{g r_1^2}{2\mu} \ln\left(\frac{r}{r_2}\right)$$

QED

d) part (b) is balance of forces & μ is included in τ term
part (c) $\frac{dp}{dz}$ is set by combination of μ_{oil} & $\bar{u}_{oil}$.

Examiners' report for 1B Paper 4 Thermofluid Mechanics 2004

Section B

Question 4 – Flow Mixing

91% of students attempted the question, the average mark was 57.1%.

This is a question of a similar type to those asked over the last few years. The most common major error that was made by around 40% of students was to try to calculate the pressure rise in the mixing region using either the Bernoulli equation or the Bernoulli "with loss" equation. This was of concern to the examiner. Several thought that the change in the flux of mechanical energy was given by Δp_0 rather than $Q\Delta p_0$, where Q is the volume flow rate. Almost everyone managed to answer part (a) correctly.

Question 5 – Non-dimensional analysis of boat drag

25% of students attempted the question, the average mark was 58.3%.

This question is very similar to material covered in the lecture notes and on the examples papers. The question tended to be well answered or barely attempted. The most common error was to simplify the relationships for working with air in such a way as to prevent the calculation of viscous drag in the water. Several students did not non-dimensionalise the drag force terms in the windtunnel. There was the usual reluctance to attempt a non-dimensional analysis question, but the average mark was higher on this question than any other in section B.

Question 6 – laminar flow, thin shear layer theory

81% of students attempted the question, the average mark was 57.1%.

This question is a slightly different version of a question that has been asked for many years. The students are taught flow between parallel plates and along pipes. This question analyses flow along a pipe, but around 30% of students tried to analyse the flow as if it were between parallel plates. This meant that the areas used in the control volume were incorrect and the first proof was in error. Many could not set up a valid control volume. The descriptive parts of the question were generally answered badly, even though the same questions are asked on the examples papers.