UNIVERSITY OF CAMBRIDGE

ENGINEERING TRIPOS PART IB

Thursday 3 June 2004 2 to 4

Paper 6

INFORMATION ENGINEERING

*Answer not more than **four** questions.*

*Answer at least **one** question from each section.*

All questions carry the same number of marks.

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

There are no attachments to this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

(TURN OVER

SECTION A

*Answer at least **one** question from this section*

1 Consider the feedback system of Fig. 1.

(a) Show that the output is given by

$$\bar{y}(s) = \frac{KG(s)}{1+KG(s)}\bar{r}(s) + \frac{G(s)}{1+KG(s)}\bar{d}(s)$$

where

$$G(s) = \frac{1}{s(s+K_1)} .$$

[8]

(b) For this system the error is defined as $e(t) = r(t) - y(t)$.

(i) Determine the steady state error when the disturbance is a unit step, i.e., $d(t) = H(t)$ and $r(t) = 0$. [3]

(ii) Show that for a ramp input, i.e., $r(t) = bt$ and $d(t) = 0$, the steady state error is bK_1/K. [3]

(c) Determine suitable values for K and K_1 to ensure that the steady state error for the ramp input in (b) part (ii) is less than $0.16b$ assuming that the damping factor is equal to 0.6 and that the minimum value of K is to be used. [6]

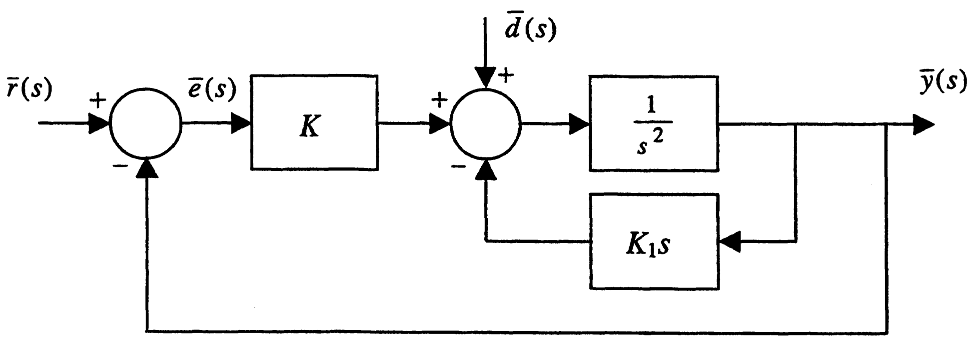

Fig. 1

2 Figure 2 shows an electric motor driving a flywheel of inertia J. The motor has negligible inertia. A disturbance torque T_d acts on the flywheel. A feedback loop is used with the aim of maintaining a constant angular velocity. A tachometer attached to the motor shaft produces a voltage $K_t\omega$, where $\omega = \dot{\theta}$ and θ is the angular position of the shaft. The voltage V_r is a reference input applied by the user. The voltage amplifier has a gain of K_a and the motor field current I_f is related to the field voltage by

$$V_f = R_f I_f + L_f \dot{I}_f$$

where R_f and L_f are constants. The motor exerts a torque T equal to $K_m I_f$.

(a) Draw a block diagram of the feedback system and find its return ratio (i.e., open loop transfer function). Show that the closed-loop transfer functions from $\overline{V}_r(s)$ to $\overline{\omega}(s)$ and from $\overline{T}_d(s)$ to $\overline{\omega}(s)$ are given respectively by

$$H_1(s) = \frac{K_m K_a}{R_f J s(1+\tau_f s) + K_m K_a K_t} \quad \text{and} \quad H_2(s) = \frac{-R_f(1+\tau_f s)}{R_f J s(1+\tau_f s) + K_m K_a K_t}$$

where $\tau_f = L_f / R_f$. [10]

(b) Taking $R_f = 1$ and $L_f = 0.1$, choose K_a (in terms of the other parameters) such that the closed-loop damping factor is at least $1/\sqrt{2}$. [6]

(c) Determine the final angular velocity of the flywheel if $V_r = \alpha$ and $T_d = 0$. What is the final angular velocity if a unit step disturbance torque $T_d = H(t)$ is also applied? [4]

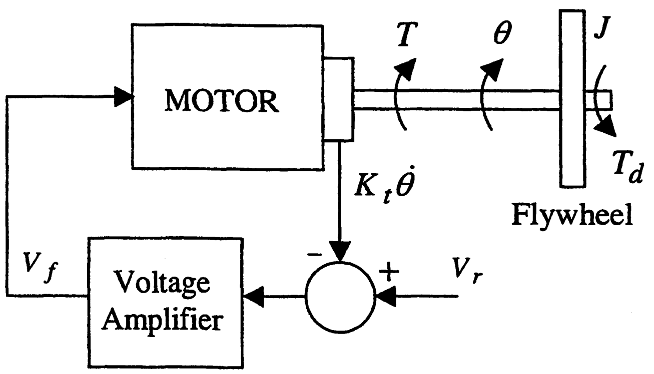

Fig. 2

3 (a) Explain the meaning of the terms gain margin and phase margin of a feedback system. [6]

(b) A control system consisting of a plant $G(s)$ and a controller $K(s)$ in a unity negative feedback configuration is shown in Fig. 3. The plant $G(s)$ has no poles in the right half plane and has the Bode diagram shown in Fig. 4. Estimate the gain margin and phase margin of the closed-loop control system when $K(s) = 4$. [6]

(c) The proportional controller in (b) is replaced with a compensator having the transfer function

$$K(s) = \frac{24(s+50)}{(s+400)} .$$

On the semilog paper supplied, sketch the Bode diagram for $K(s)$ paying particular attention to the asymptotes and corner frequencies.

Determine the new gain margin and phase margin and comment on the closed-loop behaviour for the two choices of $K(s)$. [8]

An additional copy of Fig. 4 is provided on a separate sheet and should be handed in if constructions are made on it.

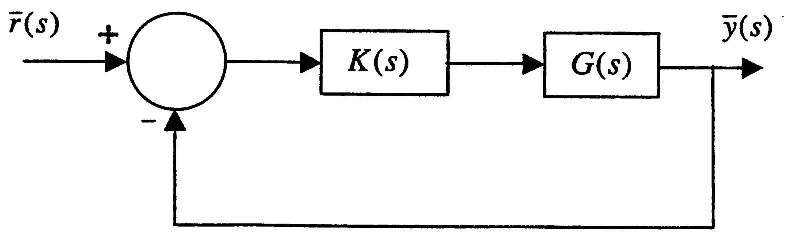

Fig. 3

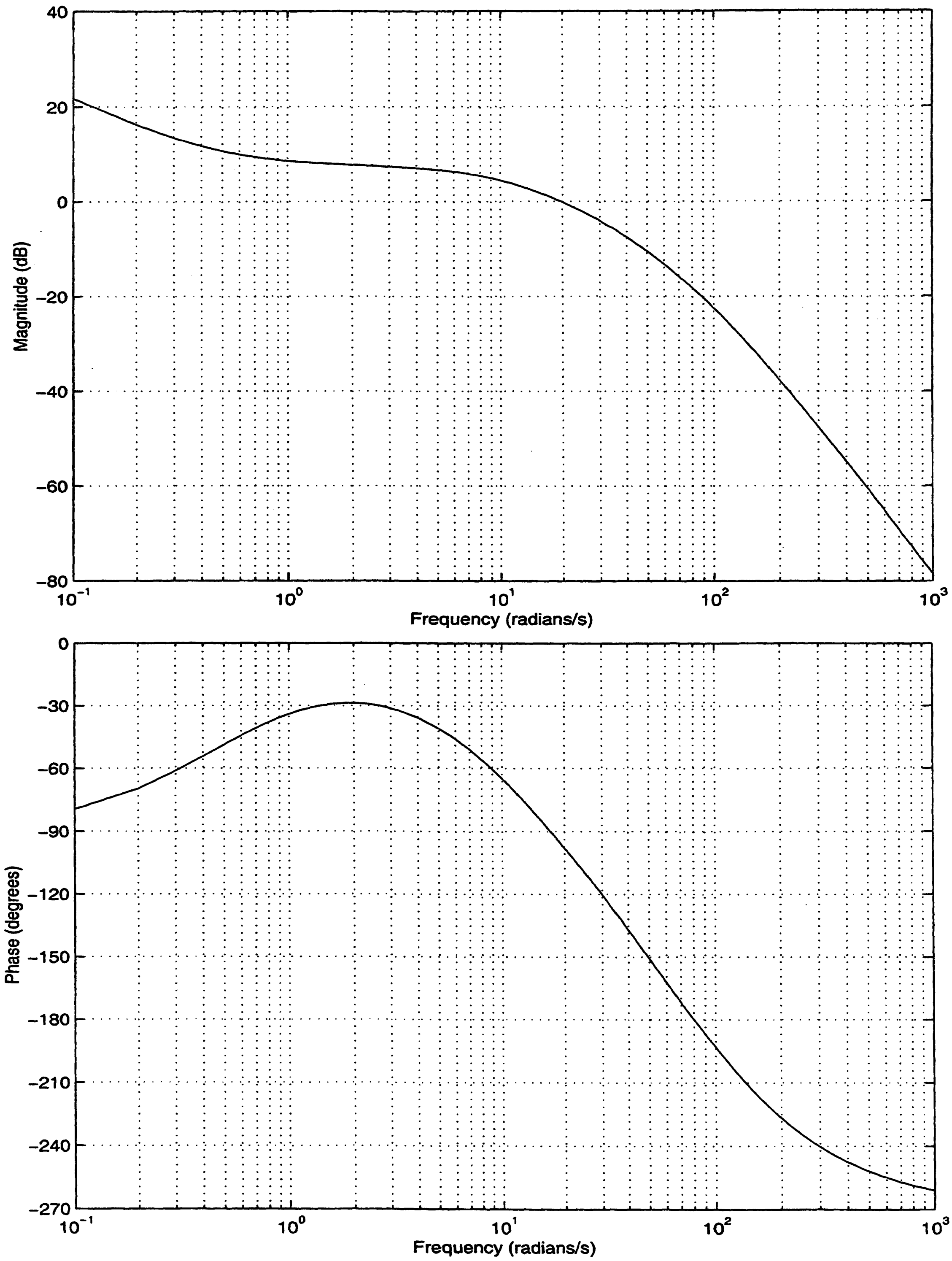

Fig. 4

(TURN OVER

4 (a) For the system shown in Fig. 5 which uses proportional negative feedback with $K(s) = k_p$, briefly describe how the information needed to plot the Nyquist diagram may be determined experimentally. What practical problems may arise?

How can the Nyquist diagram be used to determine if the closed-loop system is stable? You may assume that $G(s)$ is a stable system. [8]

(b) Part of the Nyquist diagram for a system having a transfer function

$$G(s) = \frac{5(s+3)}{s(s+5)(s+2)(s+1)}$$

is given in Fig. 6.

(i) If the phase margin of the closed-loop system is 45 degrees, show that k_p is approximately equal to 0.51 and estimate the gain margin. [4]

(ii) Estimate the maximum amplitude of the sensitivity function defined as

$$S(j\omega) = \frac{1}{1 + G(j\omega)K(j\omega)}$$

using the value of k_p determined in part (i). [2]

(c) Sketch against frequency the magnitude of the closed-loop frequency response (from $\bar{r}$ to $\bar{y}$) for the feedback system in (b), and estimate its maximum amplitude. Briefly comment on the step response of the closed-loop system. [6]

An additional copy of Fig. 6 is provided on a separate sheet and should be handed in if constructions are made on it.

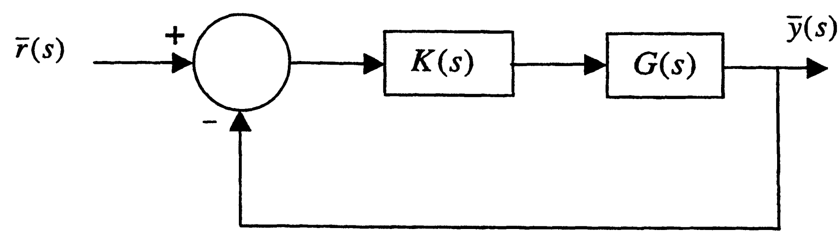

Fig. 5

(cont.

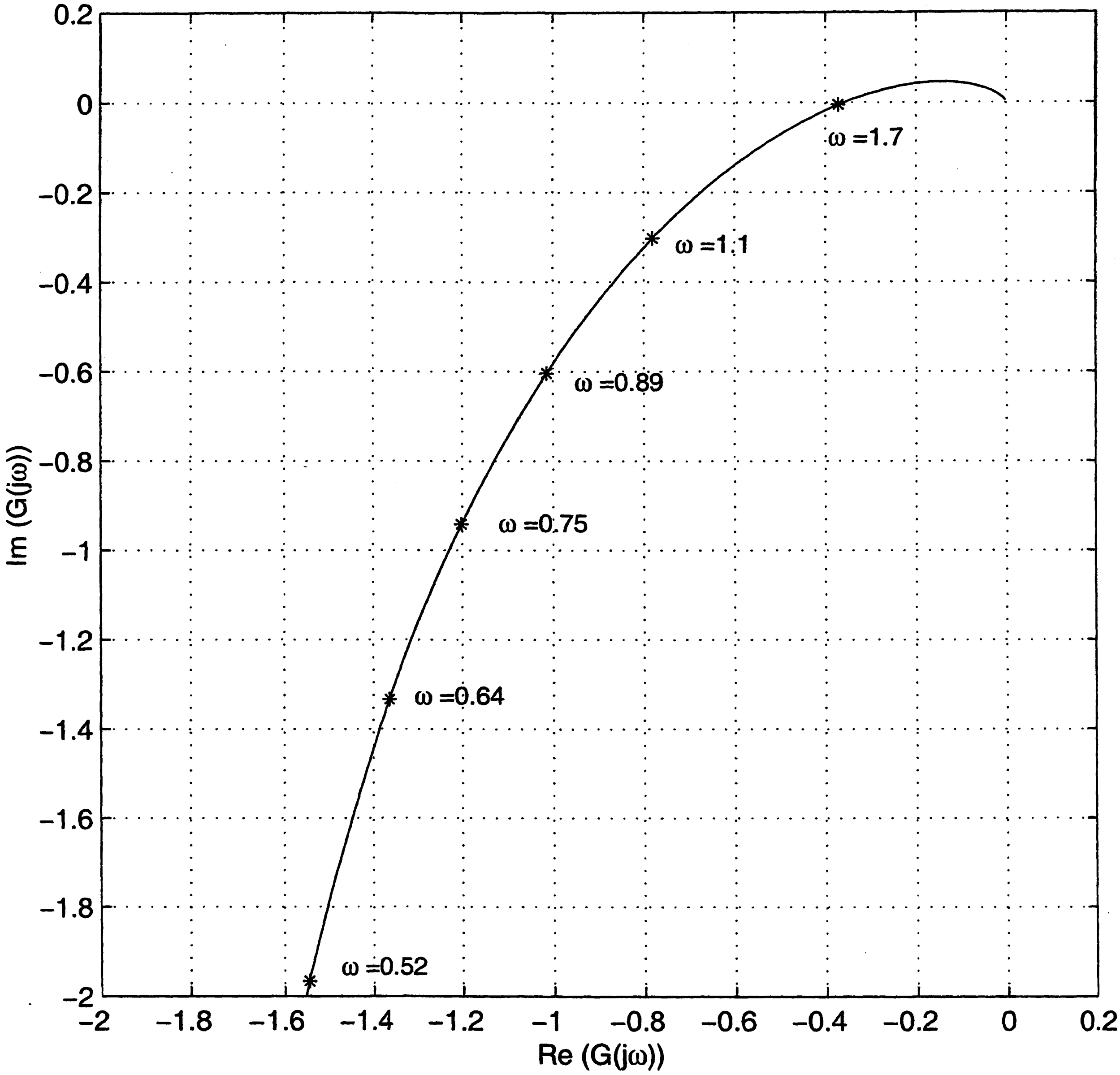

Fig. 6

(TURN OVER

SECTION B

*Answer at least **one** question from this section*

5 (a) Figure 7 shows the simplified (lumped) equivalent circuit for a long twisted-pair cable. Show that the transfer function from the input source voltage E_S to the load resistance voltage V_L is

$$H(s) = \frac{\overline{V}_L(s)}{\overline{E}_s(s)} = \frac{R_L}{4R + R_S + R_L} \times \frac{1}{(sCR_P + 1)}$$

where R_P is the parallel combination of $(2R + R_L)$ and $(2R + R_S)$.

[10]

(b) A twisted-pair cable has a loop resistance of 24 Ω/km and a shunt capacitance of 10 nF/km. Note that loop resistance is defined as the resistance measured between the two conductors at one end of the cable when those at the far-end are short-circuited. The source and load resistances are both 600 Ω.

An asymmetrical digital subscriber line (ADSL) modem produces a signal with a bandwidth of 64 kHz. What is the maximum length of the specified twisted-pair cable that could be used with the ADSL modem? [6]

(c) What is the bandwidth of a cable of length 1 km? [4]

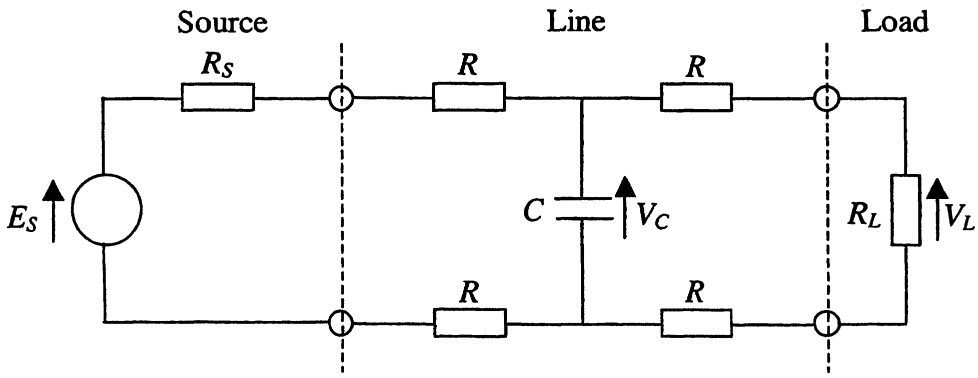

Fig. 7

6 (a) Why is modulation required for radio communication systems? [2]

(b) Double sideband amplitude modulation (DSB-AM) of a carrier at a frequency ω_c by an information bearing signal $x(t)$ is defined by the equation

$$s(t) = [a + x(t)]\cos\omega_c t \ .$$

Consider the case where the information bearing signal is a cosine wave $x(t) = b\cos\omega_m t$. Sketch the waveform $s(t)$ assuming a = 1 V, b = 0.7 V and that $\omega_c >> \omega_m$. Mark all significant voltage levels on your sketch. What is the maximum possible value of b and why? [6]

(c) A simple diode-resistor 'square-law' circuit can be made with input $x(t)$ and output $y(t)$ such that $y = x^2$. If the input to this square-law circuit is

$$x(t) = 2 + a_1(t) + a_2(t)$$

where $a_1(t) = b\cos\omega_m t$ is the modulating signal and $a_2(t) = a\cos\omega_c t$ is the carrier signal, show that $y(t)$ yields a DSB-AM signal when appropriately bandpass filtered. State the upper and lower passband frequencies for the bandpass filter and determine the modulation index of the resulting DSB-AM signal. [6]

(d) What is the maximum power efficiency of DSB-AM with a sinusoidal modulating signal? Briefly describe other forms of amplitude modulation which offer improved power efficiency. [6]

END OF PAPER