ENGINEERING TRIPOS PART IIA

Module 3C6 Examination, 2010

Answers

2 (a) u(0,t)=0, u'(L,t)=0

(b) $u(x,t)=\frac{2LF}{EA\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{(n-1/2)^2}\sin\left(\frac{(n-1/2)\pi x}{L}\right)\cos\left(\frac{(n-1/2)\pi ct}{L}\right).$

(c) $u(x,t)=\frac{2LF}{EA\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{(n-1/2)^2}\sin\left(\frac{(n-1/2)\pi x}{L}\right)\cos\left(\frac{(n-1/2)\pi ct}{L}\right)e^{-\zeta_n\omega_n t}$

3 (a) $T=\frac{2}{3}ma^2\left(\dot\theta_1^2+\dot\theta_2^2+\dot\theta_3^2+\dot\theta_4^2\right),$

$V=\frac{1}{2}ka^2\left[(\theta_1-\theta_2)^2+(\theta_2-\theta_3)^2+(\theta_3-\theta_4)^2\right]$

(c) $[1\ 1\ 1\ 1]^{\mathrm{T}},\quad \omega_1=0,\quad \omega_3=\sqrt{\frac{3k}{2m}}$

(d) $\alpha=-1\pm\sqrt{2},\quad \omega_2=0.663\sqrt{\frac{k}{m}},\quad \omega_4=1.60\sqrt{\frac{k}{m}}$, all frequencies are exact.

4 (a) $T=\frac{1}{2}m\dot x_1^2+\frac{1}{2}m\dot x_2^2+2m\dot y^2+\frac{1}{2}ma^2\dot\theta^2,\quad [M]=m\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&4&0\\0&0&0&a^2\end{bmatrix}.$

$V=\frac{1}{2}k\left[x_1^2+x_2^2+(y-a\theta-x_1)^2+(y+a\theta-x_2)^2\right]$

(b) $\alpha=\frac{3}{4}\pm\frac{\sqrt{17}}{4},\ \omega^2=0.219\frac{k}{m},2.281\frac{k}{m};\quad \beta=\pm\frac{\sqrt{2}}{a},\ \omega^2=\left(2\pm\sqrt{2}\right)\frac{k}{m}$

Final Version