ENGINEERING TRIPOS PART IIA

Thursday 6^{th} May 2010 2:30 to 4:00

Module 3A6

HEAT AND MASS TRANSFER

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper.

SPECIAL REQUIREMENTS
Engineering Data Book.
CUED approved calculator allowed.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 A pressure transducer must be placed as close as possible to a hot test section containing air at temperature T_0. The surroundings are at temperature T_∞. The transducer is mounted on a steel tube of outer diameter D_o and inner diameter D_i as shown in Fig. 1. The transducer is placed at a distance L from the test section along the tube. The tube is very long. The outer side of the tube is cooled by natural convection to the surrounding air with a constant heat transfer coefficient h_∞. The air inside the tube is stagnant. The air has a conductivity k_a. The conductivity of the steel tube is k_s. The entire system is in a steady state.

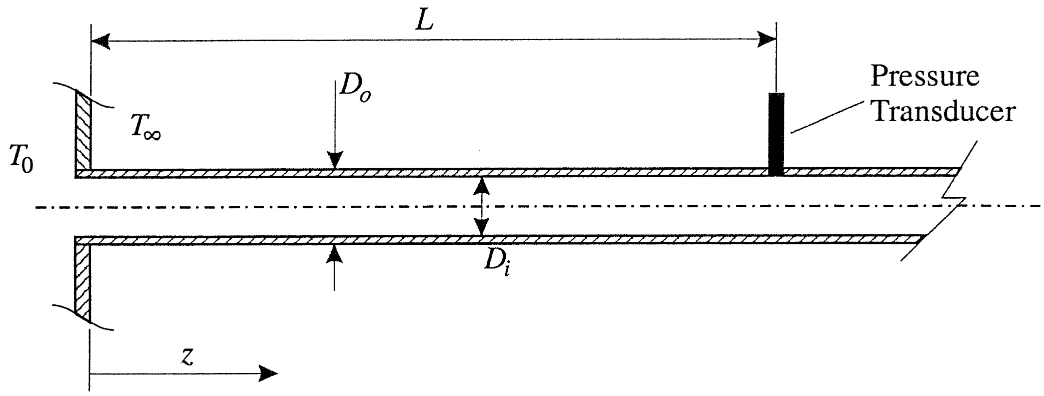

Fig. 1

(a) Assume initially that the temperature of the air in the tube is the same as the temperature T of the steel tube, which is only a function of the distance z from the test section. By considering an appropriate elemental control volume, show that the temperature of the steel is given by:

$$T(z) = T_\infty + (T_0 - T_\infty)\exp(-mz)$$

where:

$$m = \sqrt{\frac{4h_\infty}{k_s D_o (1 - (D_i / D_o)^2)}}$$

[40%]

(cont.

(b) For the case above, determine the minimum value of the heat transfer coefficient h_∞ if the maximum allowable temperature of the pressure transducer is 400 K at L = 0.10 m. Use the following values: $T_0 = 900$ K, $T_\infty = 300$ K, $D_o = 4\times10^{-3}$ m, $D_i = 3\times10^{-3}$ m, $k_s = 10$ Wm^{-1}K^{-1}.

[10%]

(c) Assume now that the steel tube is extremely thin and that the air temperature inside the tube now varies in the radial direction as well as in the axial direction. Write down a differential equation and the associated boundary conditions for the temperature of the air $T(z,r)$ inside the tube under steady-state conditions. Justify your assumptions. Do not try to solve the equation. You may use the following expression for the Laplacian operator in cylindrical coordinates:

$$\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{\partial^2}{\partial z^2} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}$$

[15%]

(d) For the conditions in (c) above, sketch the variation of the temperature along the axis of the tube. Also sketch the radial temperature profile inside the tube for several values of z.

[20%]

(e) Using the properties and parameters defined above, determine a criterion for when the temperature of the air inside the tube is approximately constant in the radial direction. Similarly, what criterion must be satisfied when the temperature across the thickness of the steel tube is approximately constant?

[15%]

(TURN OVER

2 A detector is mounted centrally over a circular disk undergoing heat treatment to monitor its surface temperature, as shown in Fig. 2. The detector area is placed parallel with the disc at a height H above the disk. The detector area A_1 is much smaller than the area $A_2 = \pi R_0^2$, where R_0 is the radius of the disk. The disk emissivity ε is a function of emission angle, wavelength and temperature, and is given by:

$$\varepsilon = \varepsilon_n \cos\theta$$

where

$$\varepsilon_n = \varepsilon_0 \left[1 - \exp\left(-\frac{\alpha}{\lambda T} \right) \right]$$

and θ is the viewing angle to the normal, λ is the wavelength, T is the temperature and α is a positive constant. The surroundings are at a temperature T_∞.

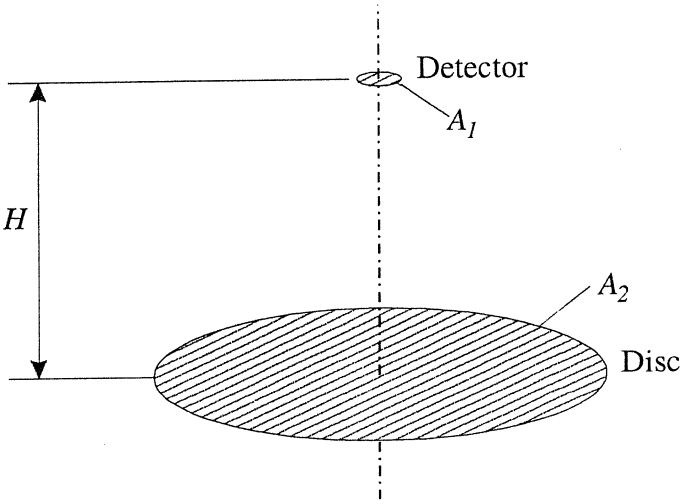

Fig. 2

(a) Neglecting radiation to or from the surroundings, show that the total irradiative flux at a given wavelength from the disk to the detector is given by:

$$G(\lambda) = \varepsilon_n I_b A_1 \frac{2\pi}{3} \left[1 - \frac{1}{[1 + (R_0 / H)^2]^{3/2}} \right]$$

where I_b is the blackbody radiation intensity.

[40%]

(cont.

(b) The blackbody radiation intensity I_b is given by:

$$I_b = \frac{1}{\pi\lambda^5} \frac{C_1}{\exp(C_2/(\lambda T)) - 1}$$

where C_1 and C_2 are constants. The sensor is fitted with a narrowband filter which allows only wavelengths close to λ_0 to pass through it. Find an expression for the ratio of the total radiation received by the detector at temperature T relative to that received at a reference temperature T_0.

[15%]

(c) Explain how the sensor above could be used to measure the temperature of the disc.

The disc temperatures range between 300 K and 1500 K. The constants have values of: $C_2 = 1.4388 \times 10^4\,\mu\text{m K}$ and $\alpha = 1000\,\mu\text{m K}$. Narrowband filters are available for wavelengths from 1 μm to 10 μm. Discuss the merits of using the higher, an intermediate or the lower filter wavelengths.

[25%]

(d) Show that the contribution of the radiative exchange with the surroundings can be neglected if the surroundings are at a temperature that is much lower than that of the disc. Assume that the radiating elements have grey body characteristics.

[20%]

(TURN OVER

3. A light hydrocarbon fuel is introduced as a gas at a pressure p_0 into the headspace of a closed vessel containing a heavy oil (Fig. 3). The fuel then begins to diffuse into the oil. The depth of the oil is so large that the concentration of fuel at the bottom of the vessel remains negligible throughout the process. The headspace has a constant volume V. The temperature T of the contents of the vessel is uniform and constant during the process. The gas constant of the fuel is R.

At the surface of the oil, it can be assumed that the concentration of the fuel in the oil is in equilibrium with the gas according to:

$$c_s = Sp$$

where c_s is the mass concentration of the fuel at the surface of the liquid, S is the solubility constant and p is the pressure of the light hydrocarbon fuel.

Assume that the fuel concentration in the oil is very low, that the oil vapour pressure is negligible and that at the moment when the fuel is introduced, the oil contains no light hydrocarbon species.

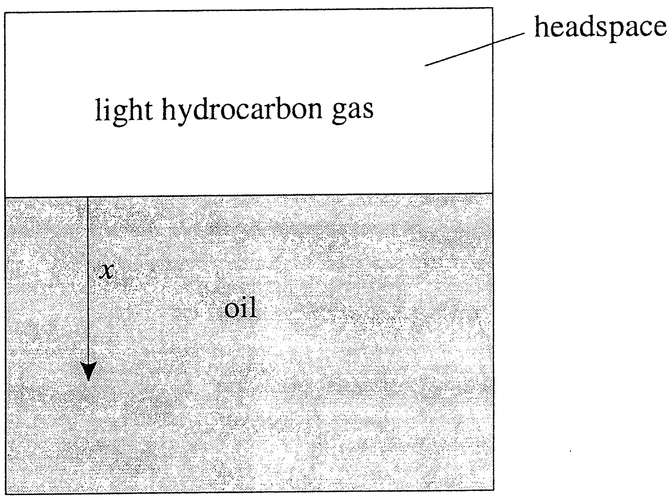

Fig. 3

(cont.

(a) The diffusion of the light hydrocarbon fuel into the oil is assumed to be one-dimensional. Show that the equation governing the evolution of the mass concentration of the fuel in the oil c, as a function of depth x and time t, can be written as:

$$\frac{\partial c}{\partial t} = D\frac{\partial^2 c}{\partial x^2}$$

where D is a constant. Justify any assumptions.

[35%]

(b) Show that if:

$$\eta = \frac{x}{2\sqrt{Dt}}$$

then the equation for the evolution of mass concentration derived above reduces to:

$$\frac{d^2 c}{d\eta^2} + 2\eta\frac{dc}{d\eta} = 0$$

[20%]

(c) Assuming that the partial pressure of the fuel in the headspace is constant in time, solve the above equation for the appropriate boundary conditions to show that the fuel concentration in the oil is given by:

$$c(\eta) = Sp_0[1 - \mathrm{erf}\,(\eta)]$$

where:

$$\mathrm{erf}\,(\eta) = \frac{2}{\sqrt{\pi}}\int_0^{\eta} \exp(-u^2)du \qquad \text{and} \qquad \mathrm{erf}\,(+\infty) = 1$$

[25%]

(d) By considering the mass flux of the fuel at the surface of the oil, derive an equation for $(1/p)(dp/dt)$, which is the fractional rate of change of the pressure in the headspace, in terms of the given variables and constants. Under what conditions can the headspace pressure be considered as constant?

[20%]

(TURN OVER

4 A fluid has a density ρ, a specific heat capacity at constant pressure c_p and a thermal conductivity k, all of which are constant.

The fluid is flowing between two parallel plates that are a distance $2a$ apart. The plates are heated so that there is a uniform rate of heat transfer per unit area $\dot{q}$ from each plate to the fluid. The flow is steady, laminar and fully developed. The temperature profile is fully developed so that the gradient of temperature in the direction of the flow does not vary across the flow.

(a) Show that:

$$\dot{q} = \rho\, a\, U_B\, c_p \frac{dT_B}{dx}$$

where U_B is the magnitude of the bulk mean velocity of the flow, T_B is the bulk mean temperature of the flow and x is the distance in the direction of the flow.

[20%]

(b) In the absence of the effects of gravity and the thermal effects of viscous work, the temperature T of the fluid is given by the differential equation:

$$\rho\, c_p \left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} \right) = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)$$

where u and v are the velocity components corresponding to the Cartesian rectangular coordinates x and y respectively.

For this problem, explain why the above differential equation can be reduced to:

$$\frac{\dot{q}}{ak}\frac{u}{U_B} = \frac{\partial^2 T}{\partial y^2}$$

[30%]

(cont.

(c) The Nusselt number is based on the dimension a and the difference between the temperature T_a at the surface of the plates and the temperature T_0 at the midpoint between the plates at the same value of x. By considering the scales involved, show that the value of Nusselt number is of the order of unity.

[10%]

(d) The velocity of the flow is given by:

$$\frac{u}{U_B} = \frac{3}{2}\left(1 - \left(\frac{y}{a}\right)^2\right)$$

where $y = 0$ corresponds to the midpoint between the plates. Show that the temperature of the flow T and the temperature of the surface of the plates T_a at the same value of x are related by:

$$T_a - T = \frac{3}{2}\frac{\dot{q}}{ak}\left(\frac{5}{12}a^2 - \frac{y^2}{2} + \frac{y^4}{12a^2}\right)$$

[30%]

(e) Determine the value of the Nusselt number as defined in part (c).

[10%]

END OF PAPER

ENGINEERING TRIPOS PART IIA 2010

ANSWERS: MODULE 3A6: HEAT AND MASS TRANSFER

Q1: (b) 1.4 W/m^2K

Q4: (e) 1.6