ENGINEERING TRIPOS PART IIA

Monday 2 May 2011 9 to 10.30

Module 3D4

STRUCTURAL ANALYSIS AND STABILITY

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachment: Data sheet for Question 2.

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
Single-sided script paper	Engineering Data Book
Graph paper	CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

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1 A thin-walled cross-section is made from sheet of uniform thickness t, density ρ, elastic modulus E and has the profile consisting of two semicircles of radius R as shown in Fig. 1.

(a) Determine the principal second moments of area, and the angle between the major axis and the x-axis. [50%]

(b) Explain why the shear centre of this cross section is located at its centroid. No computation is required. [10%]

(c) The section is mounted as a horizontal cantilever of length L with the y-axis pointing upwards. It is loaded by its own weight. Determine the maximum axial stress occurring in the cross-section. [40%]

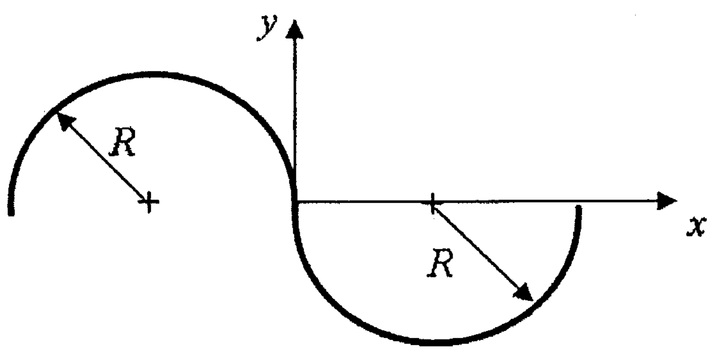

Fig. 1

2 The structure shown in Fig. 2 consists of three beams rigidly connected at point B. Joints A and C are pinned and joint D is built-in. All three beams have flexural stiffness $EI = 5 \times 10^4\,\mathrm{kNm^2}$ and their axial stiffness EA is assumed to be infinite. Any degradation of flexural stiffness due to geometric effects of axial compression may be ignored. The beam AB is loaded with uniform loading $w = 10\,\mathrm{kNm^{-1}}$.

(a) Determine the number of degrees of kinematical and statical indeterminacy of the structure. [20%]

(b) Find the bending moments in the three beams at sections adjacent to point B. (Auxiliary data sheet is attached.) [55%]

(c) Without detailed calculation, make clear sketches of two influence lines. Both should correspond to a transverse rolling point load on AC. The first should be the influence line for the vertical reaction at support A, and the second should be the influence line for the vertical reaction at support D. [25%]

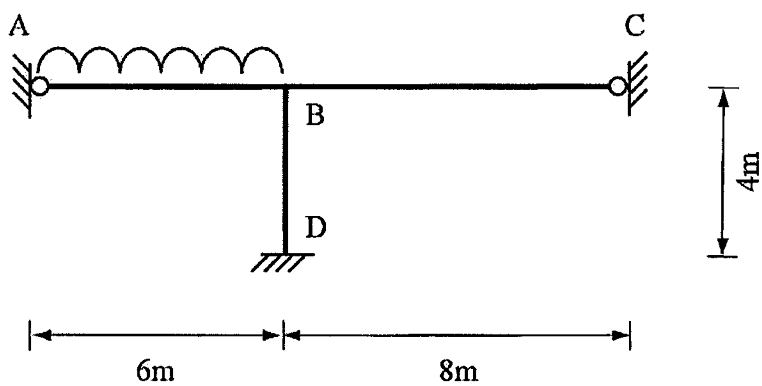

Fig. 2

3 (a) Explain briefly the physical significance of the entries in the j-th column of a stiffness matrix, in terms of the applied forces and displacements. [10%]

(b) Explain briefly the physical significance of the eigenvalues and eigenvectors of a stiffness matrix, and how knowledge of these may be useful in a stability analysis. [20%]

(c) A sub-frame containing a column BE is illustrated in Fig. 3. All members of the sub-frame are identical, having the same length L and the same flexural rigidity EI for bending within the plane of the paper. All members are rigidly connected to each other and all supports are pinned, except A which is fixed.

(i) A table of s and c stability functions is provided overleaf. Determine the effective length of the column BE for elastic flexural buckling within the plane of the paper. [30%]

(ii) A vertical load P and a couple M are applied at joint B as shown in Fig. 3. Assuming linear elastic behaviour, determine the rotations at B and C when the applied load $P = \pi^2 EI/L^2$ and the couple $M = PL/100$. Sketch the bending moment diagram and the deflected shape. [40%]

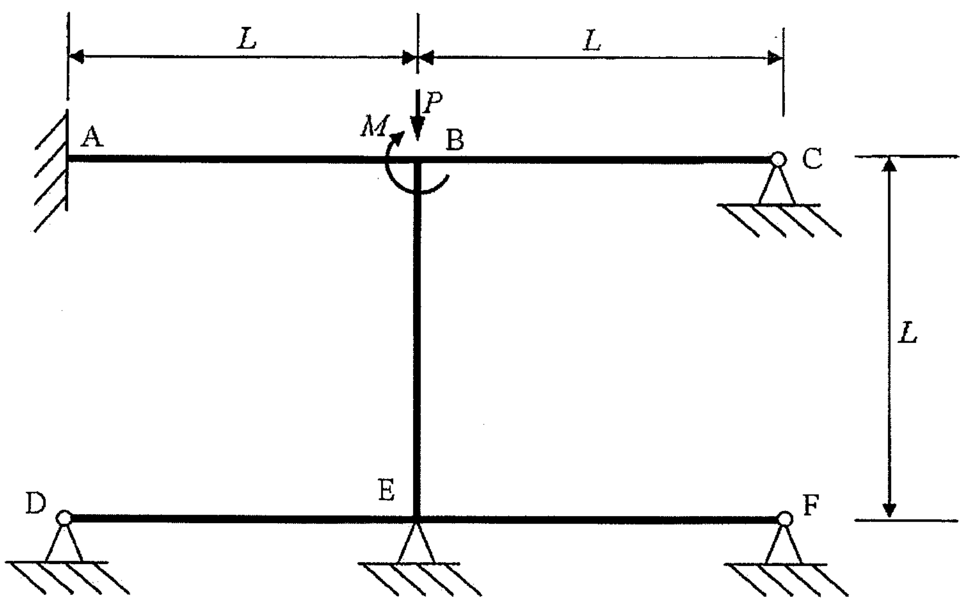

Fig. 3

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(cont.

P/P_E	s	c
0.0	4.0000	0.5000
0.2	3.7297	0.5550
0.4	3.4439	0.6242
0.6	3.1403	0.7136
0.8	2.8159	0.8330
1.0	2.4674	1.0000
1.2	2.0901	1.2487
1.4	1.6782	1.6557
1.6	1.2240	2.4348
1.8	0.7170	4.4969
2.0	0.1428	24.6841
2.2	-0.5194	-7.5107
2.4	-1.3006	-3.3703
2.6	-2.2490	-2.2312
2.8	-3.4449	-1.7081
3.0	-5.0320	-1.4157

Table 1. s and c stability functions.

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4 (a) Stating any assumptions, derive from first principles the Perry-Robertson equation used in column design:

$$(\sigma_E - \sigma)(\sigma_Y - \sigma) = \eta \sigma_E \sigma$$

where σ is the stress at the centroid, σ_Y is the yield stress, σ_E is the Euler stress and η is a dimensionless measure of the amplitude of the initial bow. [40%]

(b) Assuming the ends of a Universal Beam are simply-supported and prevent torsional rotation but provide no warping restraint, the magnitude of the equal and opposite end moments M_{cr} that will cause lateral torsional buckling is given by

$$M_{cr} = \frac{\pi}{L}\sqrt{EIGJ}\sqrt{1 + \frac{\pi^2}{L^2}\frac{E\Gamma}{GJ}}$$

where L is the length, and EI, GJ and $E\Gamma$ are respectively the minor axis flexural rigidity, the torsional rigidity and the warping rigidity. One may approximate $E\Gamma$ as $EID^2/4$ where D is the distance between flange centroids.

(i) Explain the physical significance of the final term in the equation. [10%]

(ii) Determine M_{cr} for a 6m long beam of $406 \times 140 \times 46$ UB cross-section, assuming the end conditions are as stated in the preamble above. [30%]

(iii) If the flexural stiffness of the web is ignored, and all moment is assumed to be carried by pure tension and compression forces in the two flanges, calculate the magnitude of the applied end-moment that would cause the compression flange to undergo Euler buckling about its own major axis (this being the major axis of the flange but the minor axis of the whole beam). Explain any difference between this moment and your answer for M_{cr} in part (ii) above. [20%]

END OF PAPER

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Data Sheet for Question 2: Stiffness Matrices.

Beam type I

$$
\begin{bmatrix} P_A \\ S_A \\ M_A \\ P_B \\ S_B \\ M_B \end{bmatrix} = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} & 0 & \frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} & 0 & \frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \begin{bmatrix} u_A \\ w_A \\ \phi_A \\ u_B \\ w_B \\ \phi_B \end{bmatrix}
$$

Beam type II

$$
\begin{bmatrix} P_A \\ S_A \\ M_A \\ P_B \\ S_B \\ M_B \end{bmatrix} = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{3EI}{L^3} & -\frac{3EI}{L^2} & 0 & -\frac{3EI}{L^3} & 0 \\ 0 & -\frac{3EI}{L^2} & \frac{3EI}{L} & 0 & \frac{3EI}{L^2} & 0 \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{3EI}{L^3} & \frac{3EI}{L^2} & 0 & \frac{3EI}{L^3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} u_A \\ w_A \\ \phi_A \\ u_B \\ w_B \\ \phi_B \end{bmatrix}
$$

3D4 Structural Analysis and Stability 2010-2011
Answers

1. a) $11.369R^3t$, $1.197R^3t$, 64.1°
   c) $1.8474\rho gL^2/R$

2. a) Kinematical = 1, Statical = 4 (Other answers possible if well-argued).
   b) BA -33kNm, BC 9kNm, BD 24kNm

3. c) i) $0.6325L$
   ii) 0.0113, -0.0033

4. b) ii) 91.7kNm
   iii) 60.8kNm