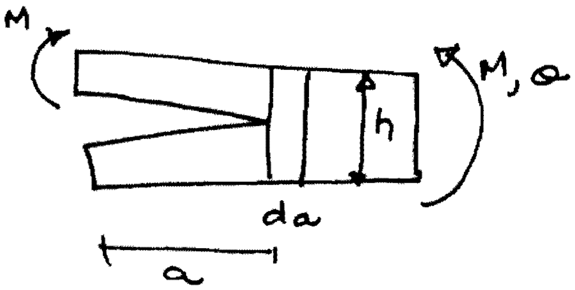
Crib 3C9 (2013)

1

(a) $G = -\frac{\partial \mathcal{U}}{\partial a}$ where $\mathcal{U}$ is the potential energy of the body & a the crack length; ie G is the change in ~~potent~~ potential energy with respect to crack length.

K is the magnitude of the stress singularity near an elastic crack tip.

(b)



$$d\mathcal{U} = -\left[\frac{1}{2}\frac{M^2}{\frac{Eh^3}{12}} - \frac{1}{2}\frac{M^2}{\frac{E}{12}\left(\frac{h}{2}\right)^3}\right]\frac{da}{B}$$

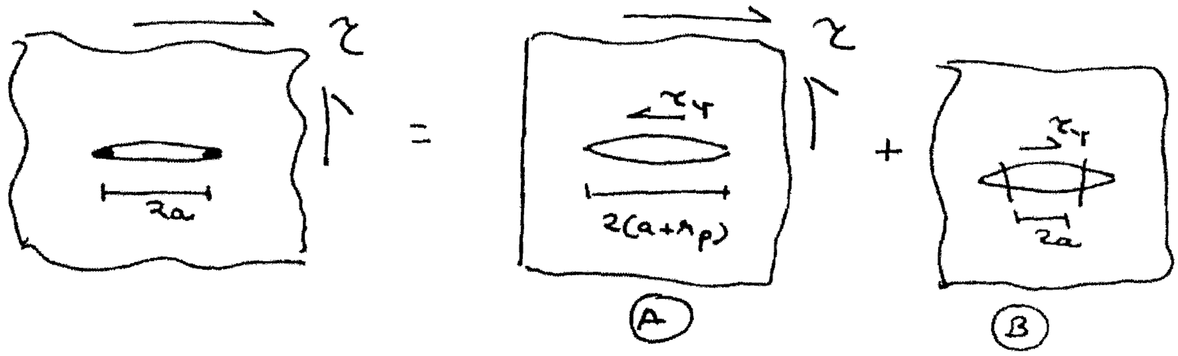
$$\frac{d\mathcal{U}}{da} = \frac{42M^2}{BEh^3}$$

$$G = \frac{1}{B}\frac{\partial \mathcal{U}}{\partial a} = \frac{42M^2}{B^2Eh^3}$$

(c) For an applied moment $\frac{dG}{da} = 0 \Rightarrow$ crack is neutrally stable.

Q2

(a)



Equivalent crack length $= 2(a + r_p)$

$$K_A = (\tau - \tau_Y)\sqrt{\pi(a+r_p)}$$

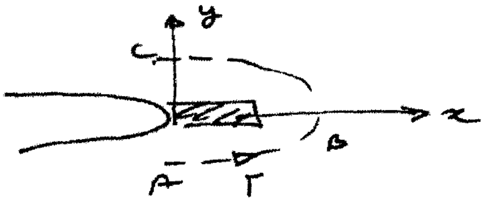
$$K_B = \frac{2}{\pi}\tau_Y\sqrt{\pi(a+r_p)}\,\sin^{-1}\left(\frac{a}{a+r_p}\right)$$

$K_A + K_B = 0 \Rightarrow$

$$(\tau - \tau_Y)\sqrt{\pi(a+r_p)} + \frac{2}{\pi}\tau_Y\sqrt{\pi(a+r_p)} = 0$$

$$\Rightarrow \frac{a}{a+r_p} = \cos\left(\frac{\pi}{2}\frac{\tau}{\tau_Y}\right)$$

(b)



$$J = \int_\Gamma w\,dy - T_i \frac{\partial u_i}{\partial x}\,ds$$

$dy = 0$ along entire contour

$ds = dx$: $T_x = -\tau_Y$ along AB

$ds = -dx$: $T_x = \tau_Y$ along BC

$$\therefore \quad J = \tau_T \int_{\Gamma_{AB}} du_x + \tau_T \int_{\Gamma_{BC}} du_x$$

$$J = \tau_T \left[u_B - u_A + u_C - u_B \right]$$

$$= \tau_T \left[u_C - u_A \right] = \tau_T \, \delta_S$$

sliding displacement

$$\Rightarrow \quad J = \tau_T \, \delta_S$$

$$\delta_S = \frac{J}{\tau_T} = \underline{\frac{K_{II}^2}{E \, \tau_T}}$$

Q3

(a) When there is a pre-existing flaw crack ~~...~~ growth occurs for $\Delta K > \Delta K_{th}$. On the other hand for a nominally smooth specimen subjected to a stress range $\Delta\sigma$, failure can only occur for $\Delta\sigma > \Delta\sigma_e$ the endurance limit.

ΔK_{th} would be used for fatigue design of a specimen wherein flaw sizes are typically greater than $\left(\frac{\Delta K_{th}}{\sigma_Y}\right)^2$ while $\Delta\sigma_e$ is used for design of components with small flaws, ie $a < \left(\frac{\Delta K_{th}}{\sigma_Y}\right)^2$

(b)

In the cyclic plastic zone the stress changes from $+\sigma_Y$ to $-\sigma_Y$ as the remote K is ~~the~~ changed from K_{max} to K_{min}

thus

$$d_{cY} = \frac{1}{\pi}\left(\frac{\Delta K}{2\sigma_Y}\right)^2$$

$$\& \quad \Delta\delta = \frac{\Delta K^2}{(2\sigma_Y)E}$$

Typically $\frac{da}{dN} \sim \frac{\Delta\delta}{2}$ & hence fatigue crack growth rates are related to the cyclic CTOD.

(c) Following a tensile overload, the plastic zone increases in size & creates a compressive residual stress field that tends to shut the crack (plasticity induced crack closure). This reduces fatigue crack growth rates.

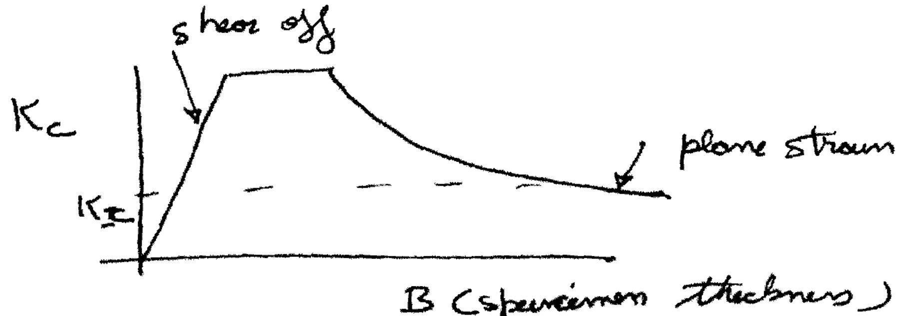
Miners rules is a linear damage summation & thus ignores the effects of plasticity induced crack closure due to tensile overloads $\Rightarrow$ Miner's rule will underestimate fatigue life (conservative).

Q4

(a) In SSY the plastic zone sizes are much smaller than an other leading dimensions of the component & LEFM & K used to model fracture.

In LSY plastic zones are large fractions of the specimen dimensions & J used to characterise fracture.

(b)



(c)

(i) $a_m = \left(\dfrac{K_{IC}}{1.135\,\sigma\sqrt{\pi}}\right)^2 = \left(\dfrac{60}{1.135\cdot 540\times\sqrt{\pi}}\right)$

$= 3.08\times10^{-3}$ m

(ii) $\dfrac{da}{dt} = 6\times10^{-6}K$

$= 1.2\times10^{-5}\,\sigma\sqrt{a}$

$= 2\left[\sqrt{a_m} - \sqrt{a_i}\right] = 1.2\times10^{-5}\,\sigma\, t_f$

$t_f = 1$ hr, $\sigma = 540$ MPa, $a_m = 3.08\times10^{-3}$ m

$\Rightarrow a_i = 2.73\times10^{-3}$ m

(iii) Proof test must cause crack > a_i to propagate immediately $\Rightarrow$

$$\sigma_p = \frac{K_{IC}}{1.13\sqrt{\pi a_i}} = 573 \text{ MPa}$$