

ENGINEERING TRIPOS PART IIA

Monday 29 April 2013 9.30 to 11.00

Module 3C9

FRACTURE MECHANICS OF MATERIALS AND STRUCTURES

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachments:

3C9 datasheet (8 pages).

STATIONERY REQUIREMENTS

Single-sided script paper

SPECIAL REQUIREMENTS

Engineering Data Book

CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 (a) Define the energy release rate G and the stress intensity factor K . Briefly describe how these quantities can be used to characterise fracture in engineering materials. [30%]

(b) Two strips of an elastic material of length L , thickness $h/2$ and depth B (into the page) are glued together using an epoxy adhesive as shown in Fig. 1. A release agent is applied to part of the surface of one of the strips to create a central crack of length $2a$. One of the strips is then cut through its thickness at the centre of the crack as shown. The entire assembly is subjected to a bending moment M . If $L \gg a \gg h$, calculate the energy release rate G (assuming symmetric crack growth) in terms of the applied loads, geometrical parameters of the beam and the Young's modulus E of the material. [50%]

(c) Discuss whether fracture in this specimen occurs in a stable or unstable manner. [20%]

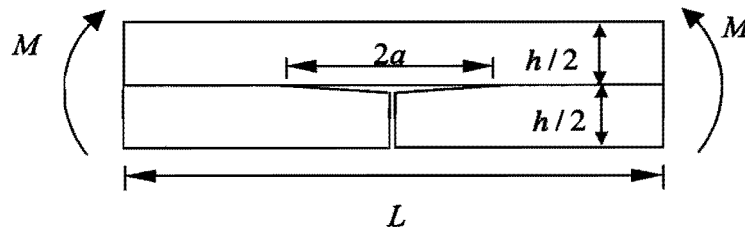


Fig. 1 (not shown to scale)

2 (a) A large metal plate with a through thickness central crack of length $2a$ is loaded in pure shear as shown in Fig. 2. The plastic zone which extends directly ahead of the crack is modelled as a slip band of length r_p within which the localised shear stress is τ_Y . Assuming plane stress conditions, estimate the length r_p using a Dugdale model. You may use the K-calibrations for crack face loading of a central crack as given in the data-sheet. [50%]

(b) Evaluate the J-integral around the crack tip plastic zone and thereby obtain an expression for the sliding displacement at the crack tip in terms of the applied mode II stress intensity factor K_{II} and τ_Y . [50%]

Note: In the usual notation the J-integral is given as $J = \int_{\Gamma} \left(W dy - T_i \frac{\partial u_i}{\partial x} ds \right)$

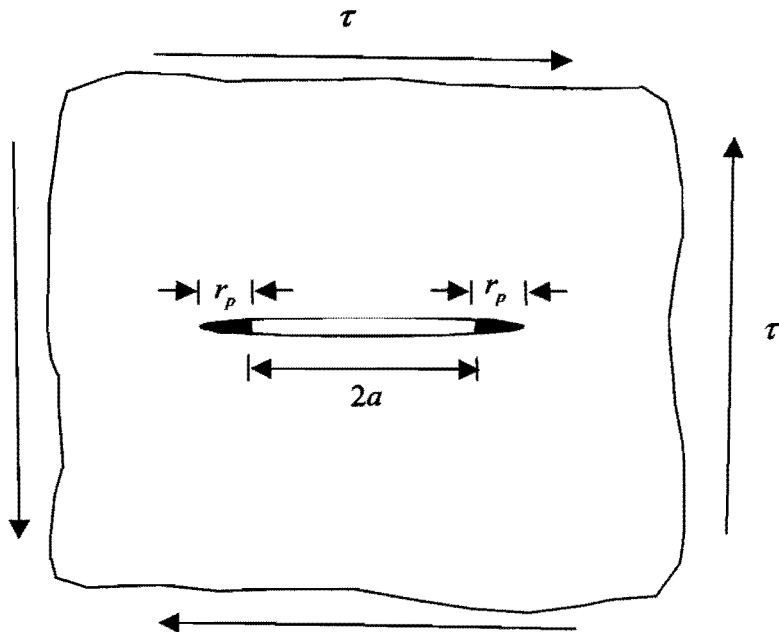


Fig. 2

(TURN OVER)

3 (a) Explain the use of threshold stress intensity range and fatigue limit in fatigue design. Give examples of types of components for which each would be the relevant fatigue design parameter. [30%]

(b) Derive an expression for the cyclic opening displacement at the tip of a fatigue crack. Why is this parameter useful in understanding the mechanism of fatigue crack growth? [40%]

(c) Explain why fatigue crack growth can arrest following a tensile peak overload. What are the implications for the use of Miner's rule in fatigue life estimation? [30%]

4 (a) Explain briefly the concepts of small scale yielding and large scale yielding. [15%]

(b) Discuss the dependence of the mode I fracture toughness upon thickness of a cracked sheet. [15%]

(c) A cylindrical pressure vessel containing hydrogen is made from a high strength steel with tensile yield strength 1800 MPa and plane-strain fracture toughness $K_{IC} = 60 \text{ MPa m}^{1/2}$. This high strength steel is known to exhibit slow growth of thumb-nail cracks in the presence of hydrogen according to the relation

$$\frac{da}{dt} = 6 \times 10^{-6} K$$

where, a is the crack depth, da/dt the crack growth rate in m hour^{-1} and K the crack tip stress intensity factor in $\text{MPa m}^{1/2}$. For a thumb-nail crack in the wall of the cylindrical pressure vessel, K is given by $K = 1.13\sigma_h\sqrt{\pi a}$ where σ_h is the hoop stress in the cylindrical pressure vessel wall. The pressure vessel is required to operate safely at a design hoop stress of 540 MPa for at least one hour. You may assume that the wall thickness of the vessel is sufficiently large so that plastic yielding is not an operative failure mode.

(i) Calculate the crack depth a_m at which the pressure vessel would immediately fail at the design hoop stress of 540 MPa. [20%]

(ii) Calculate the initial crack depth a_i which would grow to the depth a_m in one hour with the pressure vessel operating at the design hoop stress. [20%]

(iii) In order to ensure a high probability of survival of the pressure vessel for at least one hour operating at the maximum design stress, the vessel has to be proof tested before going into service. Determine the value of the proof stress. [30%]

END OF PAPER

ENGINEERING TRIPOS PART IIA

Module 3C9 – FRACTURE MECHANICS OF MATERIALS AND STRUCTURES

DATASHEET

Crack tip plastic zone sizes

$$\text{diameter, } d_p = \begin{cases} \frac{1}{\pi} \left(\frac{K_I}{\sigma_y} \right)^2 & \text{Plane stress} \\ \frac{1}{3\pi} \left(\frac{K_I}{\sigma_y} \right)^2 & \text{Plane strain} \end{cases}$$

Crack opening displacement

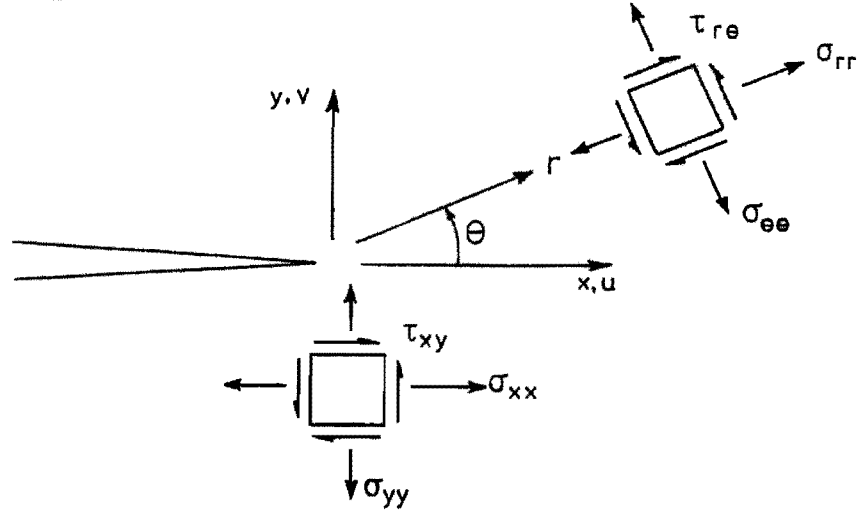
$$\delta = \begin{cases} \frac{K_I^2}{\sigma_y E} & \text{Plane stress} \\ \frac{1}{2} \frac{K_I^2}{\sigma_y E} & \text{Plane strain} \end{cases}$$

Energy release rate

$$G = \begin{cases} \frac{1}{E} K_I^2 & \text{Plane stress} \\ \frac{1-\nu^2}{E} K_I^2 & \text{Plane strain} \end{cases}$$

$$\text{Related to compliance } C: G = \frac{1}{2} \frac{P^2}{B} \frac{dC}{da}$$

Asymptotic crack tip fields in a linear elastic solid



Mode I

$$\sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left(1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right)$$

$$\sigma_{xx} = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left(1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right)$$

$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \sin \frac{\theta}{2} \cos \frac{3\theta}{2}$$

$$\sigma_{rr} = \frac{K_I}{\sqrt{2\pi r}} \left(\frac{5}{4} \cos \frac{\theta}{2} - \frac{1}{4} \cos \frac{3\theta}{2} \right)$$

$$\sigma_{\theta\theta} = \frac{K_I}{\sqrt{2\pi r}} \left(\frac{3}{4} \cos \frac{\theta}{2} + \frac{1}{4} \cos \frac{3\theta}{2} \right)$$

$$\tau_{r\theta} = \frac{K_I}{\sqrt{2\pi r}} \left(\frac{1}{4} \sin \frac{\theta}{2} + \frac{1}{4} \sin \frac{3\theta}{2} \right)$$

$$u = \begin{cases} \frac{K_I}{G} \sqrt{\frac{r}{2\pi}} \left(\frac{1-\nu}{1+\nu} + \sin^2 \frac{\theta}{2} \right) \cos \frac{\theta}{2} & \text{Plane stress} \\ \frac{K_I}{G} \sqrt{\frac{r}{2\pi}} \left(1 - 2\nu + \sin^2 \frac{\theta}{2} \right) \cos \frac{\theta}{2} & \text{Plane strain} \end{cases}$$

$$v = \begin{cases} \frac{K_I}{G} \sqrt{\frac{r}{2\pi}} \left(\frac{2}{1+\nu} - \cos^2 \frac{\theta}{2} \right) \sin \frac{\theta}{2} & \text{Plane stress} \\ \frac{K_I}{G} \sqrt{\frac{r}{2\pi}} \left(2 - 2\nu - \cos^2 \frac{\theta}{2} \right) \sin \frac{\theta}{2} & \text{Plane strain} \end{cases}$$

$$w = 0$$

Crack tip stress fields (cont'd)

Mode II

$$\sigma_{yy} = \frac{K_{II}}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \sin \frac{\theta}{2} \cos \frac{3\theta}{2}$$

$$\sigma_{xx} = -\frac{K_{II}}{\sqrt{2\pi r}} \sin \frac{\theta}{2} \left(2 + \cos \frac{\theta}{2} \cos \frac{3\theta}{2} \right)$$

$$\tau_{xy} = \frac{K_{II}}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left(1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right)$$

$$\sigma_{rr} = \frac{K_{II}}{\sqrt{2\pi r}} \left(-\frac{5}{4} \sin \frac{\theta}{2} + \frac{3}{4} \sin \frac{3\theta}{2} \right)$$

$$\sigma_{\theta\theta} = -\frac{K_{II}}{\sqrt{2\pi r}} \left(\frac{3}{4} \sin \frac{\theta}{2} + \frac{3}{4} \sin \frac{3\theta}{2} \right)$$

$$\tau_{r\theta} = \frac{K_{II}}{\sqrt{2\pi r}} \left(\frac{1}{4} \cos \frac{\theta}{2} + \frac{3}{4} \cos \frac{3\theta}{2} \right)$$

$$u = \begin{cases} \frac{K_{II}}{G} \sqrt{\frac{r}{2\pi}} \left(\frac{2}{1+\nu} + \cos^2 \frac{\theta}{2} \right) \sin \frac{\theta}{2} & \text{Plane stress} \\ \frac{K_{II}}{G} \sqrt{\frac{r}{2\pi}} \left(2 - 2\nu + \cos^2 \frac{\theta}{2} \right) \sin \frac{\theta}{2} & \text{Plane strain} \end{cases}$$

$$v = \begin{cases} \frac{K_{II}}{G} \sqrt{\frac{r}{2\pi}} \left(\frac{\nu-1}{1+\nu} + \sin^2 \frac{\theta}{2} \right) \cos \frac{\theta}{2} & \text{Plane stress} \\ \frac{K_{II}}{G} \sqrt{\frac{r}{2\pi}} \left(-1 + 2\nu + \sin^2 \frac{\theta}{2} \right) \cos \frac{\theta}{2} & \text{Plane strain} \end{cases}$$

$$w = 0$$

Mode III

$$\tau_{zx} = -\frac{K_{III}}{\sqrt{2\pi r}} \sin \frac{\theta}{2}$$

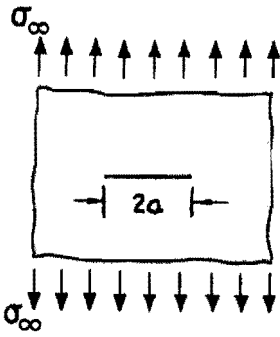
$$\tau_{yz} = \frac{K_{III}}{\sqrt{2\pi r}} \cos \frac{\theta}{2}$$

$$\sigma_{xx} = \sigma_{yy} = \sigma_{zz} = \tau_{xy} = 0$$

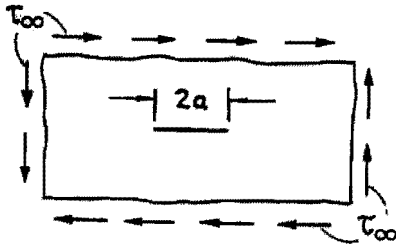
$$w = \frac{K_{III}}{G} \sqrt{\frac{2r}{\pi}} \sin \frac{\theta}{2}$$

$$u = v = 0$$

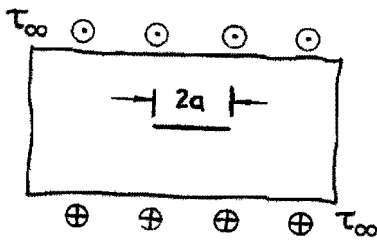
Tables of stress intensity factors



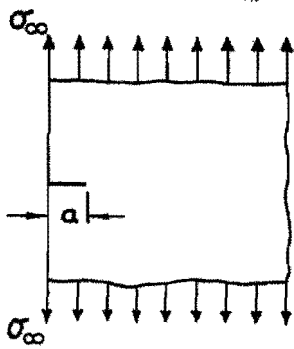
$$K_I = \sigma_{\infty} \sqrt{\pi a}$$



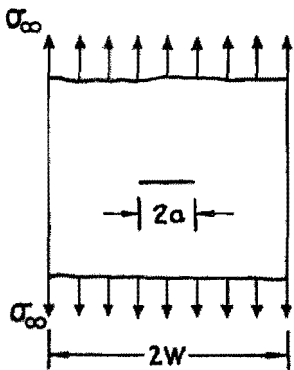
$$K_{II} = \tau_{\infty} \sqrt{\pi a}$$



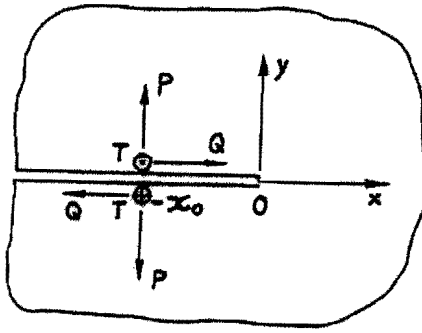
$$K_{III} = \tau_{\infty} \sqrt{\pi a}$$



$$K_I = 1.12 \sigma_{\infty} \sqrt{\pi a}$$



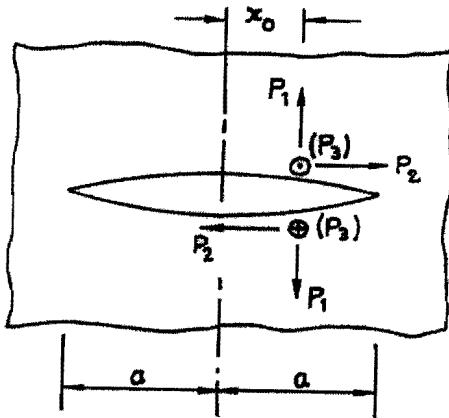
$$K_I = \sigma_{\infty} \sqrt{\pi a} \left(\frac{1 - a/2W + 0.326 a^2 / W^2}{\sqrt{1 - a/W}} \right)$$



$$K_I = \frac{2P}{\sqrt{2\pi x_0}}$$

$$K_{II} = \frac{2Q}{\sqrt{2\pi x_0}}$$

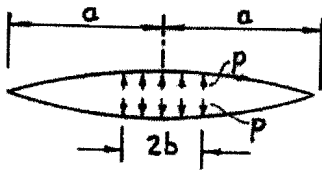
$$K_{III} = \frac{2T}{\sqrt{2\pi x_0}}$$



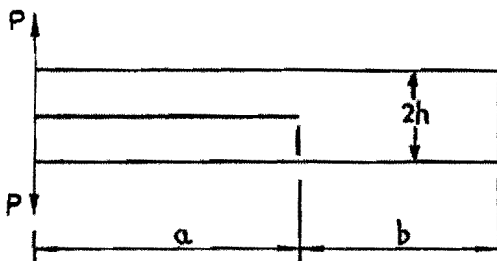
$$K_I = \frac{P_1}{\sqrt{\pi a}} \sqrt{\frac{a+x_0}{a-x_0}}$$

$$K_{II} = \frac{P_2}{\sqrt{\pi a}} \sqrt{\frac{a+x_0}{a-x_0}}$$

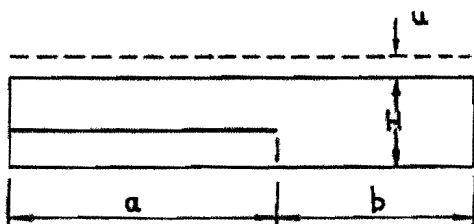
$$K_{III} = \frac{P_3}{\sqrt{\pi a}} \sqrt{\frac{a+x_0}{a-x_0}}$$



$$K_I = \frac{2pb}{\sqrt{\pi a}} \frac{a}{b} \arcsin \frac{b}{a}$$

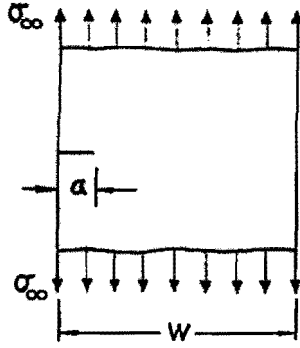


$$K_I = \frac{2\sqrt{3}}{h\sqrt{h}} \frac{Pa}{B} \quad h \ll a \text{ and } h \ll b$$



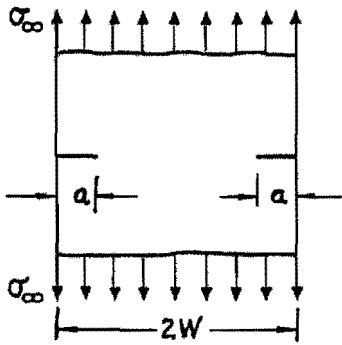
$$K_I = \sqrt{\frac{1}{2\alpha H}} Eu \quad H \ll a \text{ and } H \ll b$$

$$\alpha = \begin{cases} 1-\nu^2 & \text{Plane stress} \\ 1-3\nu^2-2\nu^3 & \text{Plane strain} \end{cases}$$

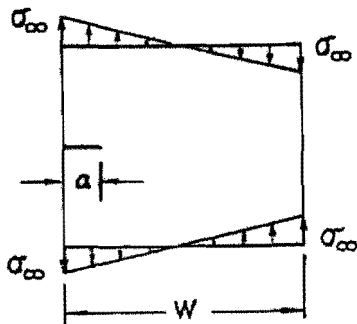


$$a/W < 0.7$$

$$K_I = \sigma_{\infty} \sqrt{\pi a} \left(1.12 - 0.23 \frac{a}{W} + 10.6 \frac{a^2}{W^2} - 21.7 \frac{a^3}{W^3} + 30.4 \frac{a^4}{W^4} \right)$$

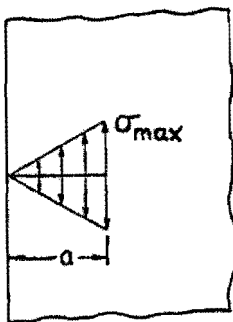


$$K_I = \sigma_{\infty} \sqrt{\pi a} \left(\frac{1.12 - 0.61 a/W + 0.13 a^3/W^3}{\sqrt{1 - a/W}} \right)$$

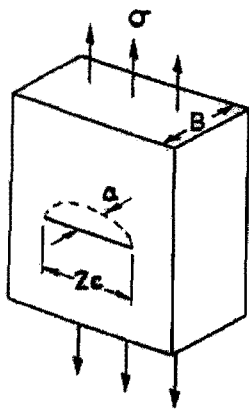


$$a/W < 0.7$$

$$K_I = \sigma_{\infty} \sqrt{\pi a} \left(1.12 - 1.39 \frac{a}{W} + 7.3 \frac{a^2}{W^2} - 13 \frac{a^3}{W^3} + 14 \frac{a^4}{W^4} \right)$$

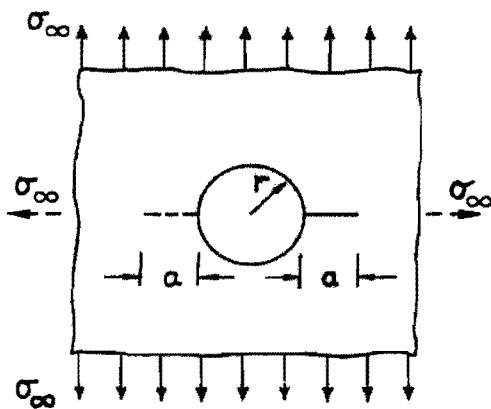
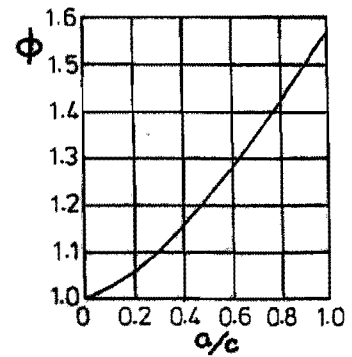


$$K_I = 0.683 \sigma_{\max} \sqrt{\pi a}$$



$$K_I = \frac{1.12}{\Phi} \sigma \sqrt{\pi a}$$

$$\Phi = \int_0^{\pi/2} \left(1 - \frac{c^2 - a^2}{c^2} \sin^2 \theta \right)^{\frac{1}{2}} d\theta$$

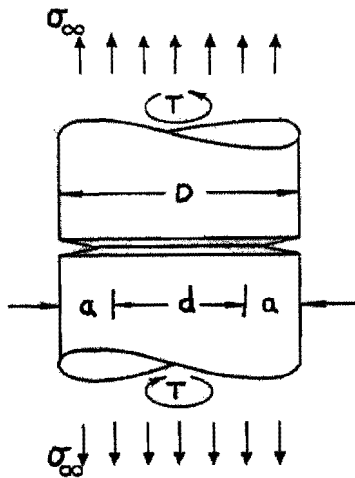


$$K_I = \sigma_{\infty} \sqrt{\pi a} F\left(\frac{a}{r}\right)$$

value of $F(a/r)^{\dagger}$

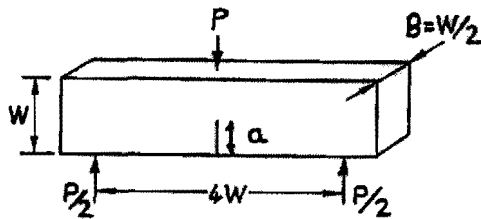
$\frac{a}{r}$	One crack		Two cracks	
	U	B	U	B
0.00	3.36	2.24	3.36	2.24
0.10	2.73	1.98	2.73	1.98
0.20	2.30	1.82	2.41	1.83
0.30	2.04	1.67	2.15	1.70
0.40	1.86	1.58	1.96	1.61
0.50	1.73	1.49	1.83	1.57
0.60	1.64	1.42	1.71	1.52
0.80	1.47	1.32	1.58	1.43
1.0	1.37	1.22	1.45	1.38
1.5	1.18	1.06	1.29	1.26
2.0	1.06	1.01	1.21	1.20
3.0	0.94	0.93	1.14	1.13
5.0	0.81	0.81	1.07	1.06
10.0	0.75	0.75	1.03	1.03
∞	0.707	0.707	1.00	1.00

$\dagger U = \text{uniaxial } \sigma_{\infty} \quad B = \text{biaxial } \sigma_{\infty}.$

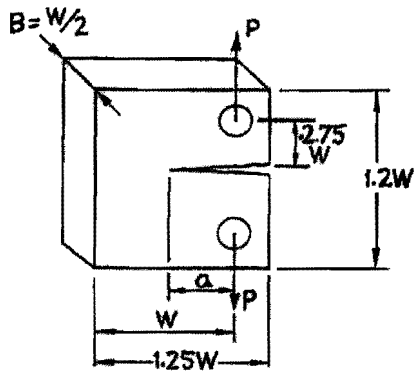


$$K_I = \sigma_{\infty} \sqrt{\pi a} \left(\frac{D}{d} + \frac{1}{2} + \frac{3}{8} \frac{d}{D} - 0.36 \frac{d^2}{D^2} + 0.73 \frac{d^3}{D^3} \right) \frac{1}{2} \sqrt{\frac{D}{d}}$$

$$K_{III} = \frac{16T}{\pi D^3} \sqrt{\pi a} \left(\frac{D^2}{d^2} + \frac{1}{2} \frac{D}{d} + \frac{3}{8} + \frac{5}{16} \frac{d}{D} + \frac{35}{128} \frac{d^2}{D^2} + 0.21 \frac{d^3}{D^3} \right) \frac{3}{8} \sqrt{\frac{D}{d}}$$



$$K_I = \frac{4P}{B} \sqrt{\frac{\pi}{W}} \left\{ 16 \left(\frac{a}{W} \right)^{1/2} - 2.6 \left(\frac{a}{W} \right)^{3/2} + 12.3 \left(\frac{a}{W} \right)^{5/2} - 21.2 \left(\frac{a}{W} \right)^{7/2} + 21.8 \left(\frac{a}{W} \right)^{9/2} \right\}$$



$$K_I = \frac{P}{B} \sqrt{\frac{\pi}{W}} \left\{ 16.7 \left(\frac{a}{W} \right)^{1/2} - 104.7 \left(\frac{a}{W} \right)^{3/2} + 369.9 \left(\frac{a}{W} \right)^{5/2} - 573.8 \left(\frac{a}{W} \right)^{7/2} + 360.5 \left(\frac{a}{W} \right)^{9/2} \right\}$$

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