EGT2
ENGINEERING TRIPOS PART IIA

Thursday 21 April 2016 9.30 to 11

Module 3A6

HEAT AND MASS TRANSFER

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 Consider the steady flow of a fluid with dynamic viscosity ν and diffusivity α through a two-dimensional channel of length L and height $2H$. The fluid enters the channel at a bulk velocity U_b from a large reservoir with temperature T_∞. The channel walls are kept at a temperature T_0. The streamwise coordinate is x, the cross-stream coordinate is y. The symmetry line is at $y = 0$, and the inlet of the channel is at $x = 0$.

(a) Sketch the viscous and thermal boundary layer thicknesses, δ_ν and δ_α, as a function of x, for values of Pr = $\frac{\nu}{\alpha}$ much smaller than, much larger than and equal to unity. [15%]

(b) Consider now a case where $H \ll L$, and where $\mathrm{Pr} \ll 1$, such that the velocity profile can be approximated as $\mathbf{u} = U_b\mathbf{i}$, where $\mathbf{i}$ is the axial unit vector.

(i) Starting from the steady energy equation

$$\rho c_p \mathbf{u} \cdot \nabla T = \lambda \nabla^2 T$$

where ρ, c_p and λ are density, specific heat at constant pressure and thermal conductivity, respectively, show that the governing equation for the non-dimensional temperature $\theta = \dfrac{T - T_0}{T_\infty - T_0}$ is given as:

$$U_b \frac{\partial \theta}{\partial x} = \alpha \frac{\partial^2 \theta}{\partial y^2}$$

Justify all steps. [25%]

(ii) Show by substitution that for an initial condition $\theta(0, y) = \cos(\frac{\pi y}{2H})$ the solution to the equation above is given by $\theta = \exp(-\frac{\pi^2}{4\,\mathrm{Pe}}\frac{x}{H})\cos(\frac{\pi}{2}\frac{y}{H})$, where Pe is a Peclet number. Obtain an expression for Pe as a function of the original parameters given, and provide an interpretation for Pe. [30%]

(iii) Obtain an expression for the normalised bulk temperature $\theta_b = \frac{1}{2H}\int_{-H}^{H} \theta\, dy$ along x. [10%]

(iv) For the solution above, determine the Nusselt number $\mathrm{Nu} = \dfrac{qH}{\lambda(T_b - T_0)}$ along the channel walls, where q is the heat transfer rate at the wall, λ the thermal conductivity, and T_b the bulk temperature. [20%]

2 (a) Consider a reservoir containing a liquid of volume V, in contact with a gaseous species A at the surface of area S. Species A is convected from the gas with bulk concentration $c_{g,\infty}$ into the liquid, according to a convection coefficient h_g. The liquid is assumed to be well-stirred, so that a uniform molar concentration c_l of species A exists throughout the volume. The molar concentration of gas at the interface is $c_{g,0} = Kc_l$, where K is an equilibrium constant. Species A is destroyed by reaction with the liquid according to $w = k_R c_l$, in moles per unit volume, where k_R is a constant. The concentration of the diluent liquid is assumed to be large enough so as to remain unaffected by reaction. The initial concentration of species A in the liquid is zero. Obtain an expression for the concentration of species A in the liquid as a function of time t. [30%]

(b) Consider now the same situation, but the concentration of species A in the liquid is now controlled by the one-dimensional diffusion rate through the liquid, according to a diffusion coefficient D. Assume a coordinate system where $x = 0$ at the interface between liquid and gas.

(i) Starting from a differential species balance along x, obtain a differential equation for the evolution of c_l in the liquid as a function of x and time t, and the constants given above. [20%]

(ii) For a steady state case with reaction, finite h_g, and a concentration of $c_{l,\infty} = 0$ far from the surface, obtain an expression for the concentration c_l as a function of x, and the constants defined above. [15%]

(iii) Show by substitution that the solution for the case with zero reaction rate and $c_l(\infty,t) = 0$, and $c_l(0,t) = c_{l,0}$ for $t > 0$ is

$$c_l = c_{l,0}\left[1 - \mathrm{erf}\left(\frac{x}{2\sqrt{Dt}}\right)\right]$$

where $\mathrm{erf}(\eta) = \int_0^{\eta} \exp(-\zeta^2)\, d\zeta$. Sketch the solution. [25%]

(iv) Sketch the expected solution for the case in (b)(iii), but for a non-zero reaction rate. Do not attempt to solve the full equation. [10%]

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3 A rod of thermal conductivity λ as shown in Fig. 1 is exposed to a high temperature environment T_h along the length L_h , and to a lower temperature environment T_c along the length L_c . The convection coefficient h is constant along the rod. The hot and cold areas do not communicate except through the rod, which has a perimeter P and cross sectional area S . The system is in steady state.

(a) Assuming uniform temperature T across the rod, obtain an expression for the temperature difference $T - T_\infty$ on the hot and cold sides, where T_∞ is the temperature of the environment. Assume that the hot and cold ends at $x = -L_h$ and $x = L_c$ are at temperatures T_h and T_c , respectively. Express your answer as a function of the hot and cold temperatures, the temperature of the interface, T_0, rod lengths, and $m = \sqrt{\frac{hP}{\lambda S}}$. [35%]

(b) Determine the condition for which a steady-state solution is possible, and thus determine the temperature T_0 for this condition. [20%]

(c) The Biot number is defined as the ratio of the rate of heat transfer by convection to that of conduction across the rod. Using scaling analysis, determine a suitable expression for a characteristic Biot number as a function of the parameters given. [15%]

(d) For a value of the Biot number larger than unity, as in the case of a thick rod, the assumption in (a) breaks down. Sketch the corresponding isotherms and isoflux lines for the heat transfer through the rod under steady state conditions for such a case, assuming that the ends of the rod are still kept at the high and low temperatures indicated in (a). Annotate your diagram with important features and symmetry lines as necessary, for the case of $T_0 = 0.5(T_h + T_c)$ and $L_h = L_c$. [30%]

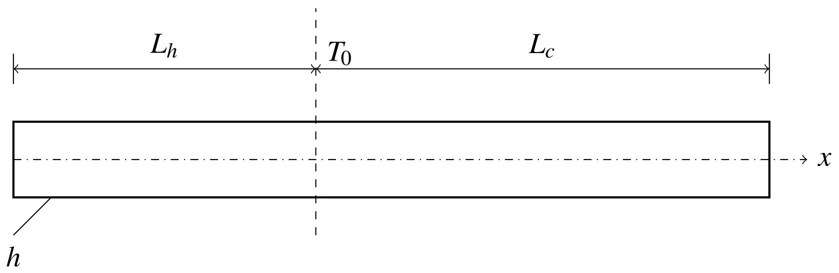

Fig. 1

4 A long triangular duct with equilateral walls 1 metre wide, as shown in Fig. 2, has radiatively gray walls. The emissivities of the walls are $\varepsilon_1 = 0.33$, $\varepsilon_2 = 0.5$ and $\varepsilon_3 = 0.8$. The corresponding temperatures are $T_1 = 1000$ K, $T_2 = 700$ K and the third wall is perfectly insulated.

(a) Using the equivalent resistance method or otherwise, determine the net radiation transfer per unit surface length of duct, for surface 1. [30%]

(b) Determine the temperature T_3 , and discuss the effect of ε_3 on the result. [20%]

(c) Flow is introduced through the channel, with a convective heat transfer coefficient h = 10 W/m^2 K, and a bulk flow temperature of T_b = 280 K, leading to a net heat transfer rate from surface 1 which is 50 percent larger than in the original case. The temperature of surface 2 remains unchanged. Determine the new temperature of surface 1, and the net heat transfer of plate 2, including convection. [50%]

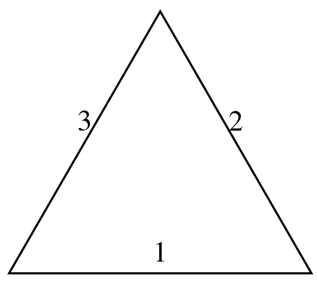

Fig. 2: Triangular duct

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