Version AHG/2

EGT2
ENGINEERING TRIPOS PART IIA

Thursday 30 April 2026   14.00 to 15.40

Module 3G4

MEDICAL IMAGING & 3D COMPUTER GRAPHICS

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

1 (a) Both ultrasound and magnetic resonance imaging (MRI) are suitable for examining soft tissue, but only the latter is normally used for brain imaging. Why? [10%]

(b) Consider the MRI imaging sequence in Fig. 1.

(i) By what name is this sequence usually known? Why are there two RF pulses? [20%]

(ii) If the main MRI field is applied in the z direction, and the patient is lying along the z axis of the machine, would this particular sequence produce an axial, sagittal or coronal slice? Justify your answer. [15%]

(iii) What is the purpose of the G_y field? [10%]

(iv) What is the purpose of the G_x field? Why is it applied twice? [20%]

(v) Explain why the detected signal's intensity is given by

$$I(x, y) \propto \mathrm{PD}(x, y) \left(1 - e^{\frac{-\mathrm{TR}}{T_1(x,y)}}\right) e^{\frac{-\mathrm{TE}}{T_2(x,y)}}$$

Hence explain how, by adjusting the values of TR and TE, it is possible to obtain PD-weighted, T_1-weighted and T_2-weighted images. [25%]

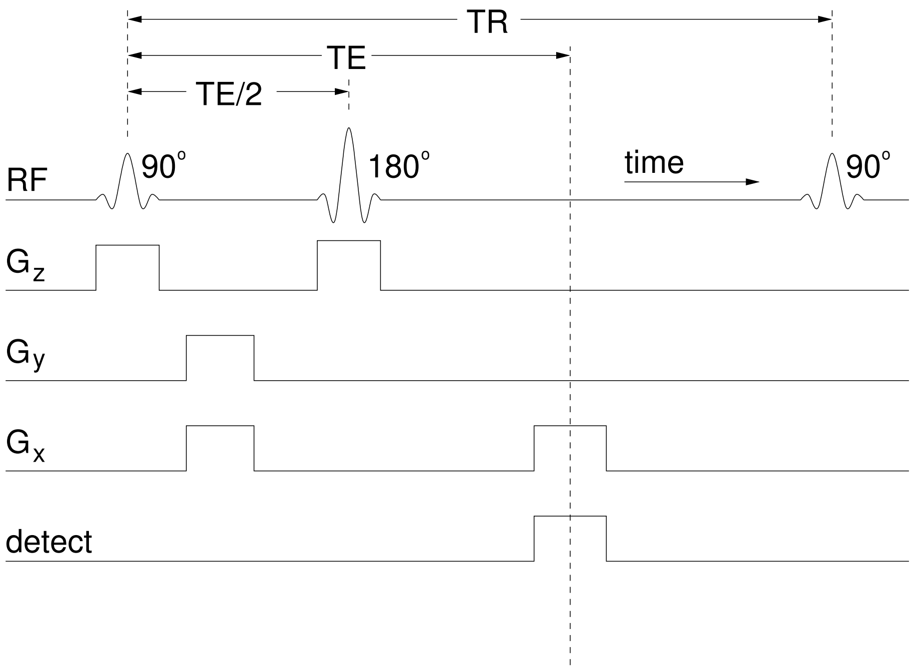

Fig. 1

(TURN OVER

2 (a) Explain the importance of, and difference between, *geometric* and *parametric* continuity in multi-segment parametric cubic spline curves. [20%]

(b) What continuity is normally achieved with B-splines and Catmull-Rom splines? Can this continuity be reduced and, if so, how? [10%]

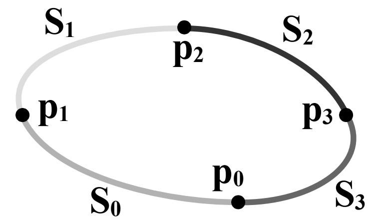

Fig. 2

(c) Figure 2 shows points $\mathbf{p}_0$ to $\mathbf{p}_3$, which are the control points of four parametric cubic spline segments $\mathbf{S}_0$ to $\mathbf{S}_3$ forming a closed loop. A novel basis matrix $\mathbf{M}$ is used to form each segment $\mathbf{S}_i$:

$$\mathbf{M} = \frac{1}{4}\begin{bmatrix} -3 & 5 & -5 & 3 \\ 6 & -9 & 6 & -3 \\ -3 & 0 & 3 & 0 \\ 0 & 4 & 0 & 0 \end{bmatrix} \qquad \mathbf{G}_i = \begin{bmatrix} \mathbf{p}_{i\ominus 1} \\ \mathbf{p}_i \\ \mathbf{p}_{i\oplus 1} \\ \mathbf{p}_{i\oplus 2} \end{bmatrix}$$

Each geometry matrix $\mathbf{G}_i$ contains four points in sequence ($\oplus$ and $\ominus$ are addition and subtraction modulo 4). It can be assumed that reversing the order of the control points in $\mathbf{G}_i$ creates an identical segment $\mathbf{S}_i^r$, but with reversed direction such that $\mathbf{S}_i^r(t) = \mathbf{S}_i(1-t)$.

(i) Show that the spline segments interpolate the control points. [15%]

(ii) What is the parametric continuity of the combined four-segment loop? [30%]

(iii) Sketch the form of the loop if the points $\mathbf{p}_0$ and $\mathbf{p}_2$ are identical, and comment on how this would affect the continuity. [15%]

(iv) Explain whether the parametric continuity in (ii) would change for a loop of more than four points based on the same matrix $\mathbf{M}$. [10%]

3 (a) Explain the difference between linear interpolation and nearest neighbour interpolation. How can a Delaunay triangulation be used to implement each of these for unstructured 2D data? [20%]

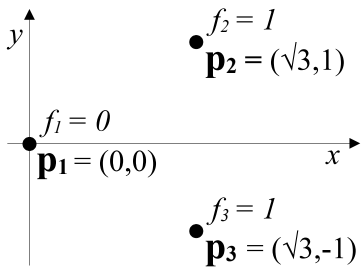

Fig. 3

(b) Figure 3 shows three points $\mathbf{p}_1$ to $\mathbf{p}_3$ with given coordinates and specified values f_1 to f_3. The values are to be interpolated using a Radial Basis Function (RBF) $s(\mathbf{p})$ with a linear basis $\phi(r) = r$, such that

$$s(\mathbf{p}) = \sum_{i=1}^{3} \lambda_i \phi(||\mathbf{p} - \mathbf{p}_i||)$$

where $r = ||\mathbf{p} - \mathbf{p}_i||$ is the Euclidean distance from $\mathbf{p}$ to $\mathbf{p}_i$. The matrix equation for solving this is given by

$$\begin{bmatrix} \phi(||\mathbf{p}_1 - \mathbf{p}_1||) & \phi(||\mathbf{p}_1 - \mathbf{p}_2||) & \phi(||\mathbf{p}_1 - \mathbf{p}_3||) \\ \phi(||\mathbf{p}_2 - \mathbf{p}_1||) & \phi(||\mathbf{p}_2 - \mathbf{p}_2||) & \phi(||\mathbf{p}_2 - \mathbf{p}_3||) \\ \phi(||\mathbf{p}_3 - \mathbf{p}_1||) & \phi(||\mathbf{p}_3 - \mathbf{p}_2||) & \phi(||\mathbf{p}_3 - \mathbf{p}_3||) \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix}$$

(i) Solve this equation for λ_1, λ_2 and λ_3, and hence write the full RBF $s(\mathbf{p})$ for the specified coordinates and values. [30%]

(ii) Comment on whether $s(\mathbf{p})$ gives the same result, within the convex hull of the points, as a linear interpolation using a Delaunay triangulation. [15%]

(iii) Write out the specific matrix equation you would need to solve if adding a first order polynomial to $s(\mathbf{p})$. Explain any new terms in this equation. [20%]

(iv) Without solving the equation in (iii), comment on whether you would expect this new RBF to affect your answers to (i) and (ii). [15%]

4 (a) Compare and contrast the process and results of Gouraud shading and Phong shading. [30%]

(b) In the context of graphics hardware, explain what is meant by a *vertex shader* and a *fragment shader*. What happens to data between the output of the vertex shader and the input to the fragment shader? [20%]

(c) The following lines of GLSL code appear in a Gouraud shader:

```
vec3 R1 = -reflect(nL1, nN);
float RdotV1 = pow(max(dot(R1, nV), 0.0), shininess);
```

What is the purpose of this code segment? What do the vectors `R1`, `nL1` and `nN` represent? Would you expect to find this code in the vertex or fragment shader? [20%]

(d) The following line of GLSL code appears in a Phong vertex shader:

```
N = normalize(mat3(modelViewMtx) * vertNormal);
```

What is the purpose of this line of code? What do you think the GLSL function `mat3()` does and why might `N` have to be normalized? [20%]

(e) The following line of GLSL code appears in a Phong fragment shader:

```
vec3 nN = normalize(N);
```

Why does `N` have to be normalized again in the fragment shader? [10%]

END OF PAPER

Part IIA 2026

Module 3G4: Medical Imaging & 3D Computer Graphics

Numerical Answers

1. (b) (ii) Axial slice
2. (c) (ii) C^2 continuity
3. (b) (i) $\lambda_1 = \frac{1}{2}$, $\lambda_2 = \lambda_3 = 0, s(\mathbf{p}) = \frac{1}{2}\phi(||\mathbf{p} - \mathbf{p}_1||) = \frac{||\mathbf{p}||}{2}$

   (iii) $$\begin{bmatrix} 0 & 2 & 2 & 1 & 0 & 0 \\ 2 & 0 & 2 & 1 & \sqrt{3} & 1 \\ 2 & 2 & 0 & 1 & \sqrt{3} & -1 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & \sqrt{3} & \sqrt{3} & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ c_0 \\ c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$